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OR THE
Whole Art of Meaſuring.

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Extraction of the SQUARE and CUBE ROOTS.

And alſo the *Multiplication of Feet and Inches*, commonly
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Meaſure all Sorts of SUPERFICIES and SOLIDS,
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THE

PREFACE.

HAVING peruſed ſeveral Books concerning the Menſuration of Superficies and Solids, and the Works of Artificers relating to Building; but not finding any one Book ſo perfect, as to give any tolerable Satisfaction to a Learner; and I having practiſed and taught Meaſuring for ſeveral Years, and thereby gained Experience and Knowlege in that Art, having learned ſome Things from one Author, and ſome Things from another, I began to think of digeſting my Thoughts into ſome ſuch Method as might give a Learner full Satisfaction, without being at the Charge of buying ſo many Books; and being importuned thereunto by ſome Friends, I fell to work, and at laſt brought them to that Perfection you here find in the following Work.

35

1. As to the *Decimal Arithmetic*, I have been as brief as the Matter would well bear, to make it plain.

2. As to the Multiplying of Feet and Inches, commonly called *Croſs-Multiplication*; my Method differs from that which is uſually taught in other Authors, as being (I think) much ſhorter and plainer.

3. In Meaſuring of Superficies and Solids, I have given the Demonſtration of the Rules, which I thought might be very acceptable to the Ingenious; for, indeed, I always look upon the Writeing of a Rule without a Demonſtration (in any Part of the Mathematics) to be but lame and defective; and for want of knowing the Reaſon of the Rule, a Learner may commit great Errors; beſides, when a Learner knows the Reaſon of the Rules, he may retain them better in his Memory. The Rule for meaſuring a Priſmoid and Cylindroid, I had out of Mr. *Everard's Art of Gauging*; but the Reaſon he does not ſhew; neither have I found it in any other Author; but that the Method is true, I have endeavoured to make plain.

The Demonſtration of the Rules for finding the Area of an Ellipſis and Parabola; alſo the Demonſtration of the Rules for finding the ſolid Content of the Fruſtum of a Cone and Pyramid, the Solidity of a Globe, of a Spheroid, a Parabolic Conoid, and of a Parabolic Spindle, and their Fruſtums, I had from the ingenious Mr. *Ward's Young Mathematician's Guide*; where the curious and ingenious Reader may ſee many other Demonſtrations algebraically performed. I have alſo demonſtrated the Rule for finding the Solidity of a Globe out of *Pardie's Elements of Geometry* (Book the 5th, Art. the 33d) publiſhed in *Engliſh* with many Additions, by the Reverend Dr. *Harris*, F. R. S. and the ſame is alſo done out of *Sturmius's Matheſis Enucleata*; ſo that the ingenious Reader may uſe which of thoſe Ways he likes beſt.

The Scale ſuppos'd to be uſed in all the Operations, is the Line of Numbers, commonly call'd, *Gunter's Line*, which is upon the ordinary Two-Feet, or Eighteen-Inch Rules, commonly uſed by the Carpenters, Maſons, *&c.* becauſe I thought it needleſs, as well as impertinent, to write the Uſe of Sliding-rules, or any other particular Scales, they being ſufficiently treated of by ſeveral Authors, *viz.* by the above-named Mr. *Everard*, in his *Art of Gauging* above-mentioned, where you have the Uſe of a Sliding-rule in Arithmetic, Geometry, in Meaſuring of Superficies and Solids, Gauging, *&c.* Likewiſe Mr. *Hunt* has written largely of the Uſes of his Sliding-rule, in Arithmetic, Geometry, Trigonometry, Gauging, Dialling, *&c.* There are ſeveral others who have explained the Uſe of their own Rules; ſo that the more curious Readers may find full Satisfaction in thoſe Authors.

One Thing I have omitted in the Book, which I think may not be very improperly inſerted in this Place; that is, how to find a Number upon the Line. If the Number you would find conſiſts only of Units, then the Figures upon the Line repreſent the Number ſought: Thus, if the Number be 1, 2, 3, *&c.* then 1, 2, 3, *&c.* upon the Line, repreſents the Number ſought. But if the Number conſiſts of Two Figures, that is, of Units and Tens, then the Figure upon the Rule ſtands for the Tens, and the large Diviſions ſtand for the Units; thus, if 34 were to be found upon the Line, the Figure 3 upon the Line is 30, and 4 of the large Diviſions (counted forwards) is the Point repreſenting 34; and if 340 were to be found, it will be at the ſame Point upon the Line; and if 304 were to be found, then the 3 upon the Line is 300, and four of the ſmaller Diviſions (counted forward) is the Point repreſenting 304. If the Number conſiſts of four Places, or Thouſands, then the Figure upon the Line ſtands for Thouſands, and the larger Diviſions are Hundreds, the leſſer Diviſions are Tens, and the tenth Parts of thoſe leſſer Diviſions are Units. Thus, if 2735 were to be found, then the 2 is 2000; and the 7 larger Diviſions (counted forward) is 700 more; and 3 of the leſſer Diviſions is 30 more; and half of one of the leſſer Diviſions is 5 more, which is the Point repreſenting 2735. You muſt remember, that between each Figure upon the Line there are 10 Parts, which I call the larger Diviſions; and each of thoſe large Diviſions are ſubdivided (or ſuppoſed ſo to be) into 10 other Parts, which I call the ſmaller Diviſions; and each of thoſe Parts ſuppoſed to be ſubdivided again into 10 other Parts, *&c.* You muſt alſo remember, that if 1, in the Middle of the Line ſtands only for 1, then 1 at the upper End will be 10, and 1 at the lower End will only be $\frac{1}{10}$; but if 1 at the lower End ſignifies 1, then 1 in the Middle ſtands for 10, and 1 at the upper End is 100, *&c.*

There is one Thing more which I would have my Reader to underſtand, and that is, how to find all ſuch proportional Numbers made uſe of in the Proportions about a Circle, and of a Cylinder, and in other Places; which Thing may be of good Uſe, to know how to correct a Number, which may happen to be falſe printed, or to inlarge any Number to more decimal Places, for more Exactneſs; for tho' I have mentioned what ſuch Numbers are, yet I have not ſhewn how to find them, which a Learner may be a little at a Nonplus to do; tho' they are eaſily found by the Rules there laid down. I ſhall therefore give two or three Examples, in this Place, of finding ſuch Numbers, which may enable my Reader to find out the reſt.

And, firſt, let it be required to find the Area of a Circle, whoſe Diameter is an Unit.

By the Proportion of *Van Culen*, if the Diameter be 1, the Circumference will be 3.1415926, *&c.* whereof 3.1416 is ſufficient in moſt Caſes. Then the Rule teaches to multiply half the Circumference by half the Diameter, and the Product is the Area: That is, multiply 1.5708 by 5 (*viz.* half 3.1416 by half 1), and the Product is .7854, which is the Area of the Circle, whoſe Diameter is 1.

Again, If the Area be required when the Circumference is 1, firſt, find what the Diameter will be, thus: As 3.1416 : to 1 : : ſo is 1 to .318309, which is the Diameter when the Circumference is 1. Then multiply half .318309 by half 1; that is .159154 by .5, and the Product is .079577, which is the Area of a Circle whoſe Circumference is 1.

If the Area be given, to find the Side of the Square equal, you need but extract the Square Root of the Area given, and it is done: So the Square Root of .7853 is .88.62, which is the Side of a Square equal when the Diameter is 1. And if you extract the Square Root of .079577, it will be .2821, which is the Side of the Square equal to the Circle whoſe Circumference is 1.

If the Side of a Square within a Circle be requir'd, if you ſquare the Semidiameter, and double that Square, and out of that Sum extract the Square Root, that ſhall be the Side of the Square which may be inſcribed in that Circle; ſo, if the Diameter of the Circle be 1, then the Half is .5; which, ſquar'd, is .25; and this, doubled, is 5, whoſe Square Root is .7071, the Side of the Square inſcribed.

Again, If the Diameter of a Globe be 1, to find the Solidity. In Sect. XI. Chap. II. it is demonſtrated, that the Globe is ⅔ of a Cylinder of the ſame Diameter and Altitude: Thus if the Cylinder's Diameter be 1, and its Altitude or Length be alſo 1, find the Solidity thereof, and take ⅔ of it, and that will be the Solidity of the Globe requir'd. Now if the Diameter be 1, the Area of the Circle, or Baſe of the Cylinder, is .7854 (as is above ſhewn), which multiplied by 1, the Altitude of the Cylinder, and the Product is alſo .7854, the Solidity of the Cylinder; ⅔ whereof is .5236, which is the Solidity of the Globe, whoſe Diameter is 1.

From what has been ſaid, the Reader may eaſily perceive how all other proportional Numbers are found, and may examine them at his Pleaſure.

I ſhall not inlarge any further upon the Matter, but leave the Book to ſpeak for itſelf; and if it prove beneficial to the ingenious Practitioners, I have my Deſire. So, wiſhing my ingenious Reader good Succeſs in his Endeavours, not doubting but he will reap Profit hereby; which that he may, is the hearty Deſire of his Well-wiſher,

W. HAWNEY.

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THE Complete MEASURER.

PART I.

CHAP. I.

Notation of DECIMALS.

A DECIMAL Fraction is an artificial Way of ſetting down and expreſſing of Natural or Vulgar Fractions, as whole Numbers: And whereas the Denominators of Vulgar Fractions are divers, the Denominators of Decimal Fractions are always certain: For a Decimal Fraction hath always for its Denominator an Unit, with a Cypher or Cyphers annexed to it, and muſt therefore be either 10, 100, 1000, 10000, *&c.* and therefore, in writing down of a Decimal Fraction, there is no Neceſſity of writing down the Denominator; for by bare Inſpection it is certainly known, it conſiſting of an Unit, with as many Cyphers annexed to it as there are Places (or Figures) in the Numerator.

Example. This Decimal Fraction $\frac{25}{100}$ may be written thus, .25, its Denominator being known to be an Unit with two Cyphers; becauſe there are two Figures in the Numerator. In like manner, $\frac{125}{1000}$ may be thus written, ·125; and $\frac{3575}{10000}$ thus, .3575; and $\frac{75}{1000}$ thus, .075; and $\frac{65}{10000}$ thus, .0065.

As whole Numbers increaſe in a decuple or tenfold Proportion, towards the Left Hand, ſo, on the contrary, Decimals decreaſe towards the Right Hand in a decuple Proportion, as in the following Scheme.

Tens of Millions.	Millions.	Hundreds of Thouſands.	Tens of Thouſands.	Thouſands.	Hundreds.	Tens.	Units.	Tenth Parts.	Hundredth Parts.	Thouſandth Parts.	Ten Thouſandth Parts.	Hundred Thouſandth Parts.	Millionth Parts, &c.
7	6	5	4	3	2	1	0	1	2	3	4	5	6

Hence it appears, that Cyphers put on the Right Hand of whole Numbers, do increaſe the Value of thoſe Numbers in a decuple (or tenfold) Proportion; but being annexed to the Right Hand of a Decimal Fraction, do neither increaſe nor decreaſe the Value thereof: So $\frac{2500}{10000}$ is equivalent to $\frac{25}{100}$ or .25. And on the contrary, though in whole Numbers, Cyphers prefixed before them, do neither increaſe nor diminiſh the Value; yet Cyphers before a Decimal Fraction do diminiſh its Value in a decuple Proportion: For .25, if you prefix a Cypher before it, becomes $\frac{025}{1000}$ or .025; and .125 is $\frac{00125}{100000}$, by prefixing two Cyphers before it, thus, .00125. And therefore, when you are to write a Decimal Fraction, whoſe Denominator hath more Cyphers than there are Figure in

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the Numerator, they muſt be ſupply'd by prefixing ſo many Cyphers before the Figures of your Numerator; as, ſuppoſe $\frac{19}{1000}$ were to be written down, without its Denominator; here, becauſe there are three Cyphers in the Denominator, and but two Figures in the Numerator, therefore prefix a Cypher before 19, and ſet it down thus, .019.

The Integers are ſeparated from the Decimals ſeveral Ways, according to Mens Fancies; but the beſt and moſt uſual Way is by a Point or Period; and if there be no whole Number, then a Point before the Fraction is ſufficient: Thus, if you were to write down 317 $\frac{217}{1000}$, it may be thus expreſſed, 317.217; and 59 $\frac{25}{10000}$ thus, 59.0025; and $\frac{75}{10000}$ thus .0075, *&c.*

CHAP. II.

Reduction of DECIMALS.

IN *Reduction of Decimals*, there are three Caſes: 1ſt, To reduce a Vulgar Fraction to a Decimal: 2dly, To find the Value of a Decimal in the known Parts of Coin, Weights, Meaſures, *&c.* 3dly, To reduce Coin, Weights, Meaſures, *&c.* to a Decimal. Of theſe in their Order.

I. *To reduce a Vulgar Fraction to a Decimal.*

The RULE.

As the Denominator of the given Fraction is to its Numerator, ſo is an Unit (with a competent Number of Cyphers annexed) to the Decimal required.

Therefore, if to the Numerator given you annex a competent Number of Cyphers, and divide the Reſult by the Denominator, the Quotient is the Decimal equivalent to the Vulgar Fraction given.

Example 1. Let $\frac{3}{4}$ be given, to be reduced to a Decimal of two Places, or having 100 for its Denominator.

To 3 (the Numerator given) annex two Cyphers, and it makes 300; which divide by the Denominator 4, and the Quotient is .75, the Decimal required, and is equivalent to $\frac{3}{4}$ given.

Note, That ſo many Cyphers as you annex to the given Numerator, ſo many Places muſt be pricked off in the Decimal found; and if it ſhall happen, that there are not ſo many Places of Figures in the Quotient, the Deficiency muſt be ſupplied, by prefixing ſo many Cyphers before the Quotient Figures, as in the next Example.

Example 2. Let $\frac{3}{573}$ be reduced to a Decimal having ſix Places.

To the Numerator, annex ſix Cyphers, and divide by the Denominator, and the Quotient is 5235; but it was required to have ſix Places, therefore you muſt prefix two Cyphers before it, and then it will be 005235, which is the Decimal required, and is equivalent to $\frac{3}{573}$.

See the Work of theſe two Examples.

```
4)3.00(.75        573)3.000000(005235
   28                     ....
  ----                ----------
   20                  1350
   20                   2040
  ----                   3210
   ..                 ---------
                          345
```

In the ſecond Example there remains 345, which Remainder is very inſignificant, it being leſs than $\frac{1}{100000}$ Part of an Unit, and is therefore rejected.

II. *To*

II. *To find the Value of a Decimal in the known Parts of Money, Weights, Meaſures,* &c.

The RULE.

Multiply the given Decimal by the Number of Parts in the next inferior Denomination, and from the Product prick off ſo many Places to the Right Hand as there were Places in the Decimal given; and multiply thoſe Figures pricked off by the Number of Parts in the next inferior Denomination, and prick off ſo many Places as before, and ſo continue to do, till you have brought it to the loweſt Denomination required.

Example 1. Let .7565 of a Pound Sterling be given to be reduced to Shillings, Pence, and Farthings.

Multiply by 20, by 12, and by 4, as the Rule directs; and always prick off four Places to the Right Hand, and you will find it make 15 *s.* 1 *d.* 2 *q.* See the Work.

$$\begin{array}{lr} & .7565 \\ & 20 \\ \hline s. & \\ & 15.1300 \\ & 12 \\ \hline d. & \\ & 1.5600 \\ & 4 \\ \hline q. & \\ & 2.2400 \end{array}$$

A more compendious Way of finding the Value of the Decimal of a Pound Sterling.

Double the firſt Figure (or Place of Primes), and it makes ſo many Shillings; and if the next Figure (or Place of Seconds) be 5, or more than 5; for the 5 add another Shilling to the former Shillings; then for every Unit in the ſecond Place count ten, and to that add the Figure in the third Place, and reckon that ſo many Farthings; but if they make above 13, abate 1; and if it be above 38, abate 2, and add the remaining Farthings to the Shillings before found.

Example 1. Let .695 of a Pound be reduced to Shillings, Pence, and Farthings.

Firſt, Double your 6, and it makes 12*s.* then take 5 out of 9, and for that reckon another Shilling, and it makes 13*s.* and the four remaining is 4 Tens, and the 5 makes 45, which being above 38, you muſt therefore caſt away 2, and there reſts 43 Farthings, which is 10*d.* ¾. So the Anſwer is 13*s.* 10*d.* ¾.

	l	*s.*	*d.*
So the Value of	.725=	14	6
And the Value of	.878=	17	6¾
And the Value of	.417=	8	4

And ſo of any other.

Let .59755 of a Pound Troy be reduced to Ounces, Peny-weights, and Grains.

Multiply by 12, by 20, and by 24, and always prick off five Places towards the Right Hand, and you will find the Anſwer to be 7 *oz.* 3 *pwt.* 13 *gr. fere.* See the Work.

.59755
12

7.17060
20

oz. *pwt.* *gr.*
3.41200 *Facit* 7 3 9.888
24

164800
82400

9.88800

Let .43569 of a Ton be reduced to Hundreds, Quarters, and Pounds.

Multiply by 20, by 4, and by 28, and the Anſwer will be 8 *C.* 2 *qrs.* 24 lb. *fere.*

.43569
20

8.71380
4 *C.* *qrs.* lb.
Facit 8 2 23.9456

2.85520
28

23.94560

Let .9595 of a Foot be reduced into Inches and Quarters

.9595
12

11.5140
4 *Facit* 11 Inches, 2 Quarters.

2.0560

III. *To reduce the known Parts of Money, Weight, Meaſure, &c. to a Decimal.*

The RULE.

To the Number of Parts of the leſſer Denomination given, annex a competent Number of Cyphers, and divide by the Number of ſuch Parts that are contained in the greater Denomination, to which the Decimal is to be brought, and the Quotient is the Decimal ſought.

Example 1. Let 6*d.* be reduced to the Decimal of a Pound.

To 6 annex a competent Number of Cyphers (ſuppoſe 3), and divide the Reſult by 240 (the Pence in a Pound), and the Quotient is the Decimal required.

```
240)6.00|0(.025
    ----
    1200
    ---- Facit .025
    ....
```

Example 2. Let 3*d.* ¾ be reduced to the Decimal of a Pound, having ſix Places.

In 3*d.* ¾ there are fifteen Farthings; therefore to 15 annex ſix Cyphers (becauſe there are to be ſix Places in the Decimal required), and divide by 960 (the Farthings in a Pound), and the Quotient is .015625.

```
96|0)15.00000|0(.015625
       .....
       -----
       540
        600
         240
          480
       -----
         ...
```

Example 3. Let 3 ¼ Inches be reduced to the Decimal of a Foot, conſiſting of four Places.

In 3 ¼ Inches, there are 13 Quarters; therefore to 13 annex four Cyphers, and divide by 48 (the Quarters in a Foot), and the Quotient is .2708.

```
48)13.0000(.2708
   ------
    340
     400
    ---
     16
```

Example 4. Let 9 *C.* 1 *qr.* 16 lb. be reduced to the Decimal of a Ton having ſix Places.

C. *qr.* lb.
9 1 16
4
——
37 *qrs.*
28
——
302 *Facit* .469642
75
——
1052 Pounds

2240)1052.00000|0(469642
......
——————
15600
21600
14400
9600
6400
——
1920

CHAP. III.

Addition of DECIMALS.

ADDITION of Decimals is performed the ſame Way as Addition of whole Numbers; only you muſt obſerve to place your Numbers right, that is, Units under Units, Primes under Primes, Seconds under Seconds, *&c.*

Example. Let 317.25, 17.125, 275.5, 47.3579, and 12.75, be added together into one Sum.

317.25
17.125
275.5
47.3579
12.75
——————
Sum 669.9829

This is ſo plain, that more Examples I think needleſs.

CHAP.

CHAP. IV.

Subtraction of DECIMALS.

SUBTRACTION of Decimals is performed likewiſe the ſame Way as in whole Numbers, reſpect being had to the right placing the Numbers (as in Addition), as in the following Examples:

	(1)		(2)
From	212.0137	From	201.1250
Subtr.	31.1275	Subtr.	5.5785
Reſts	180.8862	Reſts	195.5465
Proof	212.0137	Proof	201.1250

	(3)		(4)
From	2051.315	From	30.5
Subtr.	79.172	Subtr.	7.2597
Reſts	1972.143	Reſts	23.2403
Proof	2051.315	Proof	30.5

Note, If the Number of Places in the Decimals be more in that which is to be ſubtracted, than in that which you ſubtract from, you muſt ſuppoſe Cyphers to make up the Number of Places, as in the fourth Example.

CHAP. V.

Multiplication of DECIMALS.

MULTIPLICATION of Decimals is alſo performed the ſame Way as Multiplication of whole Numbers; but to know the Value of the Product, obſerve the Rule.

Cut off, or ſeparate by a Comma, or Prick, ſo many Decimal Places in the Product, as there are Places of Decimals in both Factors, *viz.* in the Multiplicand and Multiplier; which I ſhall farther explain in the following Examples.

Let 3.125 be multiplied by 2.75; multiply the Numbers together, as if they were whole Numbers, and the Product is 8.59375; and becauſe there were three Places of Decimals pricked off in the Multiplicand, and two Places in the Multiplier, therefore you muſt prick off five Places of Decimals in the Product, as you may ſee by the Work.

$$\begin{array}{r}
3.125 \\
2.75 \\
\hline
15625 \\
21875 \\
6250 \\
\hline
8.59375
\end{array}$$

Let 79.25 be multiplied by .459.

In this Example, becauſe two Places of Decimals are pricked off in the Multiplicand, and three in the Multiplier, therefore there muſt be five pricked off in the Product.

```
  79.25
   .459
 ------
  71325
 39625
31700
 ------
36.37575
```

Let .135272 be multiplied by .00425.

In this Example, becauſe in the Multiplicand are ſix Decimal Places, and in the Multiplier five Places, therefore in the Product there muſt be eleven Places of Decimals; but when the Multiplication is finiſhed, the Product is but 57490600, *viz.* only eight Places; thetefore, in this Caſe, you muſt prefix three Cypher; before the Product Figures, to make up the Number of eleven Places, ſo the true Product will be .00057490600.

```
     .135272
      .00425
  ----------
     676360
    270544
   541088
  ----------
.00057490600
```

More Examples for Practice.

.001472	.017532
.1045	347
7360	122724
5888	70128
14720	52596
.0001538240	6.083604
279.25	32.0752
.445	.0325
139625	1603760
111700	641504
111700	962256
124.26625	1.04244400
4.443	20.0291
15.98	35.45
35544	1001455
39987	801164
22215	1001455
4443	600873
70.99914	710.031595
7.3564	.75432
.0126	.0356
441384	452592
147128	377160
73564	226296
.09269064	.026853792

Contracted Multiplication of Decimals.

Becauſe in Multiplication of Decimal Parts, and mixed Numbers, there is no Need to expreſs all the Figures of the Product, but in moſt Caſes two, three, or four Places of Decimals will be ſufficient; therefore, to contract the Work, obſerve the following

RULE.

Write the Unit's Place of the Multiplier under that Place of the Multiplicand, whoſe Place you intend to keep in the Product; then invert the Order of all the other Figures, that is, write them all the contrary Way. Then, in multiplying, always begin at that Figure in the Multiplicand which ſtands over the Figure you are then multiplying withal, and ſet down the firſt Figure of each particular Product directly one under the other: But yet a due Regard muſt be had to the Increaſe ariſing from the Figures on the Right Hand of that Figure in the Multiplicand which you begin to multiply at. This will appear more plain by Examples.

Example 1. Let 2.38645 be multiplied by 8.2175, and let there be only four Places retained in the Decimals of the Product.

Firſt, according to the Directions, write down the Multiplicand, and under it write the Multiplier, thus; place the 8 (being the Unit's Place of the Multiplier) under 4, the fourth Place of Decimals in the Multiplicand, and write the reſt of the Figures quite contrary to the uſual Way, as in the following Work: Then begin to multiply, firſt the five which is left out, (only with regard to the Increaſe which muſt be carried from it) ſaying, 8 times 5 is 40, carry 4 in your Mind, and ſay 8 times 4 is 32, and 4 I carry is 36; and ſet down 6, and carry 3, and proceed through the reſt of the Figures, as in common Multiplication:

Then begin to multiply with 2, ſaying 2 times 4 is 8, for which I carry 1, (becauſe it is above 5) and ſay, 2 times 6 is 12, and 1 that I carry is 13; ſet down 3, and carry 1, and proceed through the reſt of the Figures: Then multiply with 1, ſaying, once 6 is 6, for which carry 1, and ſay, once 8 is 8, and 1 is 9; ſet down 9, and proceed: Then multiply with 7, ſaying, 7 times 8 is 56, for which carry 6, (becauſe it is above 55) and ſay, 7 times 3 is 21, and 6 that I carry is 27; ſet down 7, and carry 2, and proceed: Then multiply with 5, ſaying 5 times 3 is 15, for which carry 2, and ſay 5 times 2 is 10, and 2 I carry is 12, which ſet down, and add all the Products together; and the total Product will be 19.6107. See the Work.

$$\begin{array}{r}
2.38645 \\
57128 \\
\hline
19.0916 \\
4773 \\
239 \\
167 \\
12 \\
\hline
19.6107
\end{array}$$

Note, That in multiplying the Figure left out every Time next the Right Hand in the Multiplicand, if the Product be 5, or upwards to 10, you carry 1; and if it be 15, or upwards to 20, carry 2; and if 25, or upwards to 30, carry 3, *&c.*

I have here ſet down the Work of the laſt Example, wrought by the common Way, by which you may ſee both the Reaſon and Excellency of this Way, all the Figures on the Right Hand the Line being wholly omitted.

2.38645
8.2175

11|93225
167|0515
238|645
4772|90
190916|0

19.6106|52875

Example 2. Let 375.13758 be multiplied by 16.7324, ſo that the Product may have but four Places of Decimals.

Firſt, ſet 6, the Unit's Place of the Multiplier, under 5, being the fourth Place of Decimals in the Multiplicand, (becauſe four Places of Decimals were to be pricked off) and write all the reſt of the Figures backward: Then multiply all the Figures of the Multiplicand by 1, after the common Way: Then begin with the ſecond Figure of the Multiplier 6, ſaying 6 times 8 is 48, for which I carry 5, (in reſpect of the 8 left out) and 6 times 5 is 30, and 5 that I carry is 35; ſet down 5 and carry 3, and proceed after the common Method: Then begin with 7, the third Figure of the Multiplier, and ſay 7 times 5 is 35, for which carry 4, and ſay 7 times 7 is 49, and 4 I carry is 53; ſet down 3 under the firſt, and carry 5, and proceed as before: Then begin with 3, the fourth Figure of the Multiplier, and ſay 3 times 7 is 21, carry 2, and ſay 3 times 3 is 9, and 2 I carry is 11; ſet down 1 and carry 1, and proceed as before: Then begin with 2, the fifth Figure, and ſay 2 times 3 is 6, for which I carry 1, and ſay 2 times 1 is 2, and 1 I carry is three, ſet down 3; and 2 times 5 is 10; ſet down 0, and carry 1, and proceed as before: Then begin with 4 the laſt Figure of the Multiplier, and ſay 4 times 1 is 4, for which I carry nothing, becauſe it is leſs than 5;

then ſay 4 times 5 is 20; ſet down 0, and carry 2, and proceed thro' the reſt of the Figures of the Multiplicand: Then add all up together, and the Product is 6276.9520. See the Work.

375.13758 the Multiplicand.
4237.61 the Multiplier reverſed.

37513758 the Product with 1.
22508255 the Product with 6 increaſed with 6 × 8.
2625963 the Product with 7 increaſed with 7 × 5.
112541 the Product with 3 increaſed with 3 × 7.
7503 the Product with 2 increaſed with 2 × 3.
1500 the Product with 4 increaſed with 0.

6276.9520 the Product required.

Let the ſame Example be repeated, and let only one Place in Decimals be pricked off.

375.13758 the Multiplicand.
4237.61 the Multiplier inverted.

37514 the Product by 1 with the Increaſe of 1 × 7.
22508 the Product with 6 increaſed with 6 × 3.
2626 the Product with 7 increaſed with 7 × 1.
113 the Product with 3 increaſed with 3 × 5.
7 the Product with 2 increaſed with 2 × 7.
1 the Increaſe only of 4 × 3.

6276.9 the Product is the ſame as before.

More Examples for Practice.

Multiply 395.3756 by .75642; and prick off four Places in Decimals.

395.3756 the Multiplicand.
24657. the Multiplier reverſed.

2767629 the Product by 7 increaſed with 7 × 6.
197688 the Product by 5 increaſed with 5 × 5.
23722 the Product by 6 increaſed with 6 × 7.
1581 the Product by 4 increaſed with 4 × 3.
79 the Product by 2 increaſed with 2 × 5.

299.0699 the Product required.

Let the ſame Example be repeated, and let there be only one Place of Decimals.

395.3756
24657.

2767 the Product by 7 increaſed with 7 × 3.
198 the Product by 5 increaſed with 5 × 5.
24 the Product by 6 increaſed with 6 × 9 + 6 × 5.
2 the Increaſe of 4 × 9 + 4 × 3.

299.1 the Product.

Characters, and their Signification.

Note, That this Mark + ſignifies Addition; as $8+5$, that is, 8 more 5, or 8 added to 5; and $8+3+7$ denotes theſe Numbers are to be added into one Sum.

This Mark — ſignifies Subtraction, as $9-4$ ſignifies that 4 is to be taken from 9

This Mark × ſignifies Multiplication, as 7×5 ſignifies that 7 is to be multiplied by 5.

This Mark ÷ ſignifies Diviſion, as $12 \div 4$ ſignifies 12 is to be divided by 4,

This Mark = ſignifies Equality or Equation; that is, when = is placed between Numbers, or Quantities, it denotes them to be equal, as $7+5=12$, that is, 7 more 5 is equal to 12; and $15-7=8$, that is, 15 leſs by 7, is equal to 8, or ſubtract 7 from 15 and there remains 8.

This Mark : : is the Sign of Proportion, or the Golden Rule, it being always placed betwixt the two midddle Terms or Numbers in Proportion, thus, $4 : 20 :: 6 : 30$, to be thus read, as 4 is to 20, ſo is 6 to 30.

CHAP. VI.

Diviſion of DECIMALS.

DIVISION of Decimals is performed after the ſame Manner as Diviſion of whole Numbers; but to know the Value or Denomination of the Quotient, is the only Difficulty; for the reſolving of which obſerve either of the following

RULES.

RULES.

I. The firſt Figure in the Quotient muſt be of the ſame Denomination with that Figure in the Dividend which ſtands (or is to be ſuppoſed to ſtand) over the Unit's Place in the Diviſor, at the firſt ſeeking.

II. When the Work of Diviſion is ended, count how many Places of Decimal Parts there are in the Dividend more than in the Diviſor; for that Exceſs is the Number of Places which muſt be ſeparated in the Queſtion for Decimals: But if there be not ſo many Figures in the Quotient, as is the ſaid Exceſs, that Deficiency muſt be ſupplied with Cyphers in the Quotient, prefixed before the ſignificant Figures thereof, towards the Left Hand, with a Point before them; ſo ſhall you plainly diſcover the Value of the Quotient.

Theſe following Directions ought alſo to be carefully obſerved.

If the Diviſor conſiſts of more Places than the Dividend, there muſt be a competent Number of Cyphers annexed to the Dividend, to make it conſiſt of as many (at leaſt) or more Places of Decimals than the Diviſor; for the Cyphers added muſt be reckoned as Decimals.

Conſider whether there be as many decimal Parts in the Dividend as there are in the Diviſor; if there be not, make them ſo many, or more, by annexing of Cyphers.

In dividing of whole or mixed Numbers, if there be a Remainder, you may bring down more Cyphers, and by continuing your Diviſion, carry the Quotient to as many Places of Decimals as you pleaſe.

Theſe Things being conſidered, I ſhall proceed to the Practice of Diviſion of Decimals, which I ſhall endeavour to explain in as familiar and eaſy a Method as poſſible.

Example 1. Let 48 be divided by 144.

In this Example the Diviſor 144 is greater than the Dividend 48; therefore, according to the Directions above, I annex a competent Number of Cyphers, *viz.* four, with a Point between them, and divide according to the uſual Way.

```
144)48.0000(.3333
    ───────
    480
     480
      480
    ───────
       48
```

But, firſt, in ſeeking how often 144 in 48.0 (the firſt three Figures of the Dividend), I find the Unit's Place of the Diviſor to fall under the firſt Place of Decimals; therefore the firſt Figure in the Quotient is in the firſt Place of Decimals: Or, by the ſecond Rule, there being four Places of Decimals in the Dividend, and none in the Diviſor; ſo the Exceſs of decimal Places in the Dividend, above that in the Diviſor is four; ſo that when the Diviſion is ended, there muſt be four Places of Decimals in the Quotient. See the Work.

Example 2. Let 217.75 be divided by 65.

Firſt, in ſeeking how oft 65 in 217 (the firſt three Figures of the Dividend), I find the Units Place of the Diviſor to fall under the Unit's Place of the Dividend; therefore the firſt Figure in the Quotient will be Units, and all the reſt Decimals: Or, by the ſecond Rule, there being two Places of Decimals in the Dividend, and no Decimals in the Diviſor, therefore the Exceſs of Decimal Places in the Dividend, above the Diviſor, is two; ſo when the Diviſion is ended, ſeparate two Places in the Quotient, towards the Right Hand by a Point. See the Work.

65)217.75(3.35
...

227
325

...

Example 3. Let 267.15975 be divided by 13.25.

13.25)267.15975(20.163
.....

2159
8347
3975

....

In this Example, 3, the Units Place of the Diviſor, ſalls under 6, the Tens Place of the Dividend; therefore (by the firſt Rule) the firſt Figure in the Quotient is Tens: Or, by the ſecond Rule, the Exceſs of Decimal Places in the Dividend, above the Diviſor, is three; there being five Places of Decimals in the Dividend, and but two in the Diviſor, ſo there muſt be three Places of Decimals in the Quotient.

Example 4. Let 15.675159 be divided by 375.89.

375.89)15.675159(.0417
..

63955
263669

546

In this Example, 5, in the Unit's Place of the Diviſor, falls under 7, the ſecond Place of Decimals in the Dividend; therefore (by the firſt Rule) the firſt Figure in the Quotient is in the ſecond Place of Decimals; ſo that you muſt put a Cypher before the firſt Figure in the Quotient; and by the ſecond Rule, the Exceſs of decimal Places in the Dividend above the Number of decimal Places in the Diviſor is 4; for the decimal Places in the Dividend is 6, and the Number of Places in the Diviſor but two; therefore there muſt be four Places of Decimals in the Quotient; but the Diviſion being finiſhed after the common Way, the Figures in the Quotient are but three; therefore you muſt prefix a Cypher before the ſignificant Figures.

Example 5. Let 72.1564 be divided by .1347.

.1347)72.1564(535.68
...

4806
7654
9190
11080

304

In this Example, the Diviſor being a Decimal, the firſt Figure thereof falls under the Ten's Place in the Dividend; therefore the Units (if there had been any) ſhould fall under the Hundred's Place in the Dividend, and ſo the firſt Figure in the Quotient is Hundreds. And, by the ſecond Rule, there being four Places of Decimals in the Dividend, and as many in the Diviſor, ſo the Exceſs is nothing; but in dividing I put two Cyphers to the Remainders, and continue the Diviſion to two Places further; ſo I have two Places of Decimals. See the Work.

Example 6. Let .125 be divided by .0457.

```
.0457).1250000(2.735
        ....
      --------
        914
        ---
        3360
        3199
        ----
        1610
        1371
        ----
        3390
        2285
        ----
         105
```

In this Example, the Unit's Place of the Diviſor (if there had been any) would fall under the Unit's Place of the Dividend; therefore the firſt Figure of the Quotient is Units. And, by the ſecond Rule, there being ſeven Places of Decimals in the Dividend, and but four Places in the Diviſor, ſo the Exceſs is three; therefore there muſt be three Places of Decimals in the Quotient.

I ſhall ſet down only the Work of ſome few Examples more, and ſo proceed to *Contracted Diviſion.*

```
.00456).0000059791(.00131
               ..
           ------
            1419
             511
            ----
              55
```

Let it be divided by 282.

```
282)1.0000000(.0035461 ferè.
         .....
       --------
       1540
        1300
         1720
          280
```

```
.325).400000(1.2307
          ...
       --------
        750
         1000
          2500
        -------
          225
```

```
.042)495.00000(11785.71
          ......
       --------
       75
       330
        360
         240
          300
           60
        -------
           18
```

Diviſion of DECIMALS *contracted.*

IN Diviſion of Decimals the common Way, when the Diviſor hath many Figures, and it is required to continue the Diviſion till the Value of the Remainder be but ſmall, the Operation will ſometimes be large and tedious, but may be excellently contracted by the following Method.

The RULE.

By the firſt Rule of this Chapter, (*page* 20.) find what is the Value of the firſt Figure in the Quotient; then, by knowing the firſt Figure's Denomination, you may have as many or as few Places of Decimals as you pleaſe, by taking as many of the Left Hand Figures of the Diviſor as you think convenient for the firſt Diviſor; and then take as many Figures of the Dividend as will anſwer them; and in dividing, omit one Figure of the Diviſor at each following Operation. A few Examples will make it plain.

Example 1. Let 721.17562 be divided by 2.257432; and let there be three Places of Decimals in the Quotient.

2.25743

```
2.25743)721.175|62(319.467
  .....  677229
         ------
         43946
         22574
         ------
         21372
         20317
         ------
          1055
           903
         ------
           152
           135
         ------
            17
            15
           ---
             2
```

In this Example, the Unit's Place of the Diviſor ſalls under the Hundred's Place in the Dividend, and it is requir'd, that three Places of Decimals be in the Quotient, ſo there muſt be ſix Places in all, that is, three Places of whole Numbers, and three Places of Decimals. Then, becauſe I can have the Diviſor in the firſt ſix Figures of the Dividend, I cut off the 62 with a Daſh of the Pen, as uſeleſs; then I ſeek how oft the Diviſor in the Dividend, and the Anſwer is three times; put 3 in the Quotient, and multiply and ſubtract as in common Diviſion, and the Remainder is 43946. Then prick off the 3 in the Diviſor, and ſeek how often the remaining Figures may be had in 43946, the Remainder, which can be but once; put 1 in the Quotient, and multiply and ſubtract, and the next Remainder is 21372. Then prick off the 4 in the Diviſor, and ſeek how often the remaining Figures may be had in 21372, which will be 9 times; put 6 in the Quotient; multiply thus, ſaying 9 times 4 is 36, for which I carry 4, (in reſpect of the 4 laſt prick'd off) and 9 times 7 is 63, and 4 is 67; ſet down 7, and carry 6, and ſo proceed till the Diviſion be finiſh'd, always reſpecting the Increaſe made from the Figures prick'd off. Obſerve the Work, which will better inform you than many Words.

```
2.25743)721.17562(319467
                ...
       677229|
       ------|
        43946|6
        22574|3
       ------|--
        21372|32
        20316|87
       ------|---
         1055|450
          902|972
         ----|----
          152|4780
          135|4458
          ---|-----
           17|03220
           15|80201
           --|------
            1 23019
```

I have ſet down the Work of this laſt Example at large, according to the common Way, that thereby the Learner may ſee the Reaſon of the Rule; all the Figures on the Right Side the perpendicular Line being wholly omitted.

Example 2. Let 5171.59165 be divided by 8.758615; and let it be requir'd, that four Places of Decimals be prick'd off in the Quotient.

```
8.758615)5171.5916|5(590.4577
 ......
          43793075
          --------
           7922841
           7882754
           -------
             40087
             35034
             -----
              5053
              4379
              ----
               674
               613
               ---
                61
                61
                --
                ..
```

In this Example, I can't have 8, the firſt Figure in the Diviſor, in 5, the firſt Figure of the Dividend, ſo that the Unit's Place of the Diviſor falls under the Hundred's Place of the Dividend; ſo that there will be ſeven Figures in the Quotient, that is, three of whole Numbers, and four of Decimals; therefore there muſt be 7 Figures in the Diviſor, (becauſe the Number of Places in the Diviſor and Quotient will be equal) and there muſt be eight Places in the Dividend; ſo that I cut off the Figure 5 with a Daſh, as uſeleſs. Thus having proportion'd the Dividend to the Diviſor, and both to the Number of Places or Figures deſir'd in the Quotient, I proceed to divide as before, ſaying how often 8 in 51, which will be 5 times; put 5 in the Quotient, and multiply and ſubtract, and the Remainder is 7922841. Then I prick off the firſt Figure in the Diviſor, 5, and ſeek how often the remaining Figures of the Diviſor in the aforeſaid Remainder, which I find nine times; put 9 in the Quotient, and multiply thereby, ſaying 9 times 5 (the Figure prick'd off) is 45, for which I carry 5, and ſay 9 times 1 is 9, and 5 I carry is 14; ſet down 4, and carry 1, and proceed to multiply the reſt of the Figures, and ſubtract, and the Remainder will be 40087. Then prick off the Figure 1, and ſeek how often 87586 in the Remainder 40087, the Anſwer will be 0; ſo put 0 in the Quotient, and prick off the Figure 6, and ſeek how often 8758 in 40087, which will be four times; put 4 in the Quotient, and multiply, ſaying 4 times 6 (the Figure laſt prick'd off) is 24, for which I carry 2, and ſay 4 times 8 is 32, and 2 I carry is 34; ſet down 4, and carry 3; multiply the reſt of the Figures, and ſubtract as before, and ſo proceed after the ſame manner, until all the Figures of the Diviſor be prick'd off, to the laſt Figure. See the Work.

Example 3. Let 25.1367 be divided by 217.3543, and let there be five Places of Decimals in the Quotient.

In this Example, 7, the Unit's Place of the Diviſor, falls under 1, the firſt Place of Decimals; therefore the firſt Figure of the Quotient is in the firſt Place of Decimals; ſo the Quotient will be all Decimals. Then, becauſe the Quotient Figures, and the Figures of the Diviſor will be of an equal Number, daſh off the 43 in the Diviſor, and the 7 in the Dividend, as uſeleſs, and divide as before.

217.35|43

```
217.35|43)25.136|7(.11564
          21735
          -----
          3401
          2174
          ----
          1227
          1087
          ----
           140
           130
           ---
            10
             8
             -
             2
```

Altho' I have hitherto given Directions for proportioning the Diviſor and Dividend, ſo as to bring into the Quotient what Number of Decimals you pleaſe, yet there is no abſolute Neceſſity for it; but you may carry on your Diviſion to what Degree you pleaſe, before you begin to prick off the Figures of the Diviſor, in order to contract the Work, as in the following Examples, where it is not requir'd to prick off any determinate Number of Decimals, but it may be done according to Diſcretion.

2.756756

```
2.756756)7414.76717(2689.67118
  ••••••        •••
         5513512
         --------
         19012551
         16540536
         --------
         24720157
         22054048
         --------
          2666109
          2481080
          -------
           185029
           165406
           ------
            19624
            19297
            -----
              327
              276
              ---
               51
               28
               --
               23
               22
               --
                1
```

12.34254

```
12.34254)514.75498(41.705757
     ... 4937016
         -------
         2105338
         1234254
         -------
          871084
          863978
          ------
            7106
            6171
            ----
             935
             864
             ---
              71
              62
              --
               9
               8
               -
               1
```

* * *

CHAP. VII.

Extraction of the SQUARE ROOT.

If a Square Number be given;

TO find the Root thereof, that is, to find out ſuch a Number, as being multiplied into itſelf, the Product ſhall be equal to the Number given; ſuch Operation is call'd, *The Extraction of the Square Root*; which to do, obſerve the following Directions.

1ſt, You muſt point your given Number; that is, make a Point or Prick over the Unit's Place, another upon the Hundred's, and ſo upon every ſecond Figure throughout.

2dly, Then ſeek the greateſt ſquare Number in the firſt Point towards the Left Hand, placing the ſquare Number under the firſt Point, and the Root thereof in the Quotient, and ſubtract the ſaid ſquare Number from the firſt Point, and to the Remainder bring down the next Point, and call that the Reſolvend.

3dly, Then double the Quotient, and place it, for a Diviſor, on the Left Hand of the Reſolvend; and ſeek how often the Diviſor is contained in the Reſolvend, (reſerving always the Unit's Place) and put the Anſwer in the Quotient, and alſo on the Right Hand Side of the Diviſor; then multiply by the Figure laſt put in the Quotient, and ſubtract the Product from the Reſolvend, (as in common Diviſion) and bring down the next Point to the Remainder, (if there be any more) and proceed as before.

A TABLE *of Squares and Cubes, and their Roots.*

Root	1	2	3	4	5	6	7	8	9
Squ.	1	4	9	16	25	36	49	64	81
Cub.	1	8	27	64	125	216	343	512	729

Example 1. Let 4489 be a Number given, and let the ſquare Root thereof be required.

4489.67
36

127.889 Reſolvend.
889 Product.

• • •

Firſt, Point the given Number, as before directed: then, by the little Table aforegoing, ſeek the greateſt ſquare Number in 44, (the firſt Point to the Left Hand) which you will find to be 36, and 6 the Root; put 36 under 44, and 6 in the Quotient, and ſubtract 36 from 44, and there remains 8. Then to that 8 bring down the other Point 89, placing it on the Right Hand, ſo it makes 889 for a Reſolvend: then double the Quotient 6, and it makes 12, which place on the Left Hand for a Diviſor, and ſeek how often 12 in 88, (reſerving the Unit's Place) the Anſwer is 7 times; which put in the Quotient, and alſo on the Right Hand Side of the Diviſor, and multiply 127 by 7, as in common Diviſion, and the Product is 889, which ſubtracted from the Reſolvend, there remains nothing; ſo is your Work finiſh'd; and the ſquare Root of 4889 is 67; which Root if you multiply by itſelf, that is 67 by 67, the Product will be 4489, equal to the given ſquare Number, and prove the Work to be right.

Example 2. Let 106929 be a Number given, and let the ſquare Root thereof be requir'd.

```
 .  .  .
106929(.327
 9
 ------
62)169 Reſolvend.
   124 Product.
   ----
647)4529 Reſolvend.
    4529 Product.
    ----
    ....
```

Firſt, Point your given Number, as before directed, putting a Point upon the Units, Hundreds, and Tens of Thouſands; then ſeek what is the greateſt ſquare Number in 10, (the firſt Point) which by the little

Table you will find to be 9, and 3 the Root thereof; put 9 under 10, and 3 in the Quotient; then ſubtract 9 out of 10, and there remains 1; to which bring down 69, the next Point, and it makes 169 for the Reſolvend; then double the Quotient 3, and it makes 6, which place on the Left Hand of the Reſolvend for a Diviſor, and ſeek how often 6 in 16; the Anſwer is twice; put two in the Quotient, and alſo on the Right Hand of the Diviſor, making it 62. Then multiply 62 by the 2 you put in the Quotient, and the Product is 124; which ſubtract from the Reſolvend, and there remains 45; to which bring down 29, the next Point, and it makes 4529 for a new Reſolvend. Then double the Quotient 32, and it makes 64, which place on the Left Side of the Reſolvend for the Diviſor, and ſeek how oft 64 in 452, which you'll find 7 times; put 7 in the Quotient, and alſo on the Right Hand of the Diviſor, making it 647, which multiply'd by the 7 in the Quotient, it makes 4529, which ſubtracted from the Reſolvend, there remains nothing. So 327 is the Square Root of the given Number.

Example 3. Let 2268741 be a ſquare Number given, the Root whereof is requir'd.

```
           .  .  .  .
           2268741(1506.23
           1
           ------
         25)126
            125
           ------
      3006)18741
           18036
           ------
     30122)70500
           60244
           ------
   301243)1025600
           903729
           ------
Remains .121871
```

Having pointed the given Number as before directed, ſeek what is the greateſt Square Number in the firſt Point 2, which is one; put 1, the Square, under 2, and 1, the Root thereof, in the Quotient; ſubtract 1 from 2, and there remains 1; to which bring down the next Point, 26, and ſet on the Right Hand, making it 126; double the 1 in the Quotient; which makes 2; ſet 2 on the Left Hand for a Diviſor, and aſk how often 2 in 12, which will be 5 times; put 5 in the Quotient, and alſo on the Right Hand of the Diviſor, making it 25; multiply (as in common Diviſion) 25 by 5, and ſubtract the Product 125 from 126, and there remains 1. Bring down the next Point, 87, and it makes 187 for a new Reſolvend; and double the 15 in the Quotient, it makes 30 for a new Diviſor: Then ſeek how often 30 in 18, which you cannot have; ſo that you muſt put 0 in the Quotient, and alſo on the Right Hand of the Diviſor, and bring down the next Point, and it makes 18741 for another new Reſolvend: Then ſeek how often 300 in 1874, which will be 6 times; put 6 in the Quotient, and alſo on the Right Hand of the Diviſor; multiply and ſubtract, and the Remainder will be 705. Now, if you have a Mind to find the Value of the Remainder, you may annex Cyphers, by two at a time, to the Remainders, and ſo proſecute the Work to what Number of Decimal Parts you pleaſe; thus, to 705 annex two Cyphers, and it will make 70500, and the Quotient doubled, is 3012 for a Diviſor: Then ſeek how often 3012 in 7050, (rejecting the Unit's Place) which will be twice; put 2 in the Quotient, and alſo on the Right Hand of the Diviſor, and multiply and ſubtract before, and the Remainder will be as 10256; to which annex two Cyphers, and proceed as before, and you will get a 3 in the Quotient next. So the Square Root of the given Number is 1506.23, which being ſquared, or multiplied by itſelf, and the laſt Remainder added, will make the given Number as follows.

1506.23
1506.23

451869
301246
903738
7531150
150623

2268728.8129
The Remainder add — 12.1871

Proof 2268741.0000

Some more Examples for Practice.

Example 1. 7596796(2756.228 Root.
4

47)359
329

545)3067
2725

5506)34296
33036

51122)126000
110244

551242)1575600
1102484

5512448)47311600
44099584

3212016

Example 2. 751417.5745)866.4 Root.
64

166)1114
996

1726)11817
10356

17328)146157
138624

173364)753345
693456

59889

If the given Number be a mixed Number, *viz.* conſiſting of a whole Number and a Decimal together, make the Number of Decimal Places even, that is, 2, 4, 6, 8, *&c.* that ſo there may a Point fall upon the Unit's Place of the whole Numbers, as in the laſt Example, and in that following.

Example 3. Let 656714.37512 be given, to find the Square Root.

```
    656714.375120(810.379 Root.
    64
   ----
161)167
    161
   -------
16203)61437
       48609
      ------
162067)1282851
       1134469
      --------
1620749)14838220
        14586741
        --------
Remains   251479
```

In this Fxample there are five Places of Decimals; therefore put a Cypher to it, to make it even, that ſo there may a Point fall upon 4, the Unit's Place.

To find the Square Root of a Fraction.

If it be a Decimal Fraction, the Work differs nothing from the Examples aforegoing, only you muſt be mindful to point your given Number aright; for (as was before directed) the Number of Places muſt always be made even, and then begin to point at the Right Hand, as in whole Numbers.

If it be a vulgar Fraction, it muſt be reduced to a Decimal, by the firſt Rule of the ſecond Chapter.

I ſhall give an Example or two in each Caſe, and ſo conclude this Chapter.

Let .125 be a Decimal Fraction given, whoſe Square Root is required; and let it be required to have four Places of Decimals in the Root.

```
   .  .  .  .
 .12500000(.3535
   9
  ----
65)350
   325
  ----
703)2500
     2109
    -----
7065)39100
      35325
    --------
       3775
```

In this Example there muſt be five Cyphers annexed, becauſe two Places in the Square make but one in the Root.

Let the Square Root of .00715 be required.

```
  .  .  .
 007150).084
  64
 -----
164)750
     656
   -----
     94
```

In this a Cypher is added to make the Place even.

Let $\frac{7}{8}$ be a vulgar Fraction given, whoſe Square Root is required.

```
8)7000                  . . . .
  64             (.87500000(.9354
  ——                81
  60                ——
  56             183)650
  ——                 549
   40                ———
   40           1865)10100
  ——                  9325
   ..                 ————
                18704)77500
                      74816
                      —————
                       2684
```

Reduce this $\frac{7}{8}$ to a Decimal, it makes .875; to which annex Cyphers, and extract the Square Root, as if it was a whole Number. So the Root is .9354.

Let $\frac{3}{960}$ be a vulgar Fraction, whoſe Square Root is required.

```
                                 . . . .
96|0)3.000000          (.00312500(.0559 Root.
     288                  25
     ———                  ——
     120               105)625
      96                   525
     ———                   ———
     240              1109)10000
     192                    9981
     ————                   ————
      480                     19
      480
     ————
      ...
```

In extracting the Root of this, becauſe the firſt Point conſiſts of Cyphers, there muſt be a Cypher put firſt in the Quotient.

To prove this Rule, ſquare the Root, and to the Product add the Remainder, as was before directed. To ſquare a Number, is to multiply it by itſelf; and to cube it, is to multiply the Square of the Number by the Number itſelf.

CHAP. VIII.

Extraction of the Cube Root.

TO extract the Cube Root, is nothing elſe but to find ſuch a Number, as being firſt multiplied into itſelf, and then into that Product, produceth the given Number; which to perform, obſerve the following Directions.

1ſt, You muſt point your given Number, begining with the Unit's Place, and make a Point, or Dot, over every third Figure towards the Left Hand.

2dly, Seek the greateſt Cube Number in the firſt Point, towards the Left Hand, putting the Root thereof in the Quotient, and the ſaid Cube Number under the firſt Point, and ſubtract it therefrom, and to the Remainder bring down the next Point, and call that the Reſolvend.

3dly, Triple the Quotient, and place it under the Reſolvend; the Unit's Place of this under the Ten's Place of the Reſolvend; and call this the triple Quotient.

4thly, Square the Quotient, and triple the Square, and place it under the triple Quotient; the Units of this under the Ten's Place of the triple Quotient, and call this the triple Square.

5*thly*, Add theſe two together, in the ſame Order as they ſtand, and the Sum ſhall be the Diviſor.

6*thly*, Seek how often the Diviſor is contained in the Reſolvend, rejecting the Unit's Place of the Reſolvend, (as in the Square Root) and put the Anſwer in the Quotient.

7*thly*, Cube the Figure laſt put in the Quotient, and put the Unit's Place thereof under the Unit's Place of the Reſolvend.

8*thly*, Multiply the Square of the Figure laſt put in the Quotient, into the triple Quotient, and place the Product under the laſt, one Place more to the Left Hand.

9*thly*, Multiply the triple Square by the Figure laſt put in the Quotient, and place it under the laſt, one Place more to the Left Hand.

10*thly*, Add the three laſt Numbers together, in the ſame Order as they ſtand, and call that the Subtrahend.

Laſtly, Subtract the Subtrahend from the Reſolvend; and if there be another Point, bring it down in the Remainder, and call that a new Reſolvend, and proceed in all reſpects as before.

Example 1. Let 314432 be a Cubic Number, whoſe Root is required.

314432(68 Root
216

98432 Reſolvend

18 Triple Quotient of 6.
108 Triple Square of the Quotient 6.

1098 Diviſor

512 Cube of 8, the laſt Figure of the Root.
1152 The Square of 8, by the triple Quotient.
864 The triple Square of the Quotient 6 by 8.

98432 The Subtrahend

. . . . .

After you have pointed the given Number, ſeek what is the greateſt Cube Number in 314, the firſt Point, which, by the former little Table (*Page* 34.) you will find to be 216, which is the neareſt that is leſs than 314, and its Root is 6; which put in the Quotient, and 216 under 314, and ſubtract it therefrom, and there remains 98; to which bring down the next Point, 432, and annex to 98; ſo will it make 98432 for the Reſolvend. Then triple the Quotient 6, it makes 18, which write down the Unit's Place, 8, under 3, the Ten's Place of the Reſolvend: Then ſquare the Quotient 6, and triple the Square, and it makes 108, which write under the triple Quotient one Place towards the Left Hand; then add thoſe two Numbers together, and they make 1098 for the Diviſor. Then ſeek how often the Diviſor is contained in the Reſolvend, (rejecting the Unit's Place thereof) that is, how often 1098 in 9843, which is 8 times; put 8 in the Quotient, and the Cube thereof below the Diviſor, the Unit's Place under the Unit's Place of the Reſolvend: Then ſquare the 8 laſt put in the Quotient, and multiply 64, the Square thereof, by the triple Quotient 18; the Product is 1152; ſet this under the Cube of 8, the Units of this under the Tens of that: Then multiply the triple Square of the Quotient by 8, the Figure laſt put in the Quotient, the Product is 864; ſet this down under the laſt Product, a Place more to the Left Hand: Then draw a Line under thoſe three, and add them together, and the Sum is 98432, which is called the Subtrahend; which being ſubtracted from the Reſolvend, the Remainder is nothing; which ſhews the Number to be a true cubic Number, whoſe Root is 68; that is, if 68 be cubed, it will make 314432.

For, if 68 be multiplied by 68, the Product will be 4624; and this Product, multiplied again by 68, the laſt Product is 314432, which ſhews the Work to be right.

68
68
———
544
The Work 408
———
4624
68
———
36992
27744
———
The Proof 314432

Example 2. Let the Cube Root of 5735339 be required.

After you have pointed the given Number, ſeek what is the greateſt Cube Number in 5, the firſt Point, which (by the little Table, *page* 34.) you will find to be 1; which place under 5, and 1, the Root thereof, in the Quotient; and ſubtract one from 5, and there remains 4; to which bring down the next Point, it makes 4735 for the Reſolvend: Then triple the 1, and it makes 3; and the Square of 1 is 1, and the Triple thereof is 3; which ſet one under another, in their Order, and added, makes 33 for the Diviſor. Seek how often the Diviſor in the Reſolvend, and proceed as in the laſt Example.

5735339(179 Root.
1

4735 Reſolvend.

3 The Triple of the Quotient 1 the firſt Figure.
3 The triple Square of the Quotient 1.

33 The Diviſor.

343 The Cube of 7, the ſecond Figure of the Root.
147 The Square of 7 multipl. in the triple Quot. 3.
21 The triple Square of the Quot. multiplied by 7.

3913 The Subtrahend

822339 The new Reſolvend.

51 The Triple of the Quot. 17, the two firſt Fig.
867 The triple Square of the Quotient 17.

8721 Diviſor.

729 The Cube of 9, the laſt Figure of the Root.
4131 The Square of 9 multip. by the trip. Quot. 51.
7803 The triple Square of the Quotient 867 by 9.

822339 The Subtrahend.

....

In this Example, 33, the firſt Diviſor ſeems to be contained more than ſeven times in 4735, the Reſolvend, but if you work with 9, or 8, you will find that the Subtrahend will be greater than the Reſolvend.

Some more Examples for Practice.

32461759(319 The Root.
27

5461 Reſolvend.

9 The Triple of 3.
27 The triple Square of 3.

279 The Diviſor.

1 The Cube of 1 the ſecond Figure.
9 The triple Quotient by the Square of 1.
27 The triple Square, multipl. by 1, the 2d Fig.

2791 The Subtrahend.

2670759 A new Reſolvend.

93 The Triple of 31.
2883 The triple Square of 31.

28923 The Diviſor.

729 The Cube of 9, the laſt Figure.
7533 The Square of 9, by 93 the trip. Quotient.
25947 The triple Square 2883 by 9.

2670759 The Subtrahend.

. . .
84604519(439 The Root.
64

20604 Reſolvend.

12 The Triple of 4.
48 The triple Square of 4.

492 The Diviſor.

27 The Cube of 3.
108 The Square of 3, by the triple Quotient.
144 The triple Square by 3.

15507 The Subtrahend.

5097519 The Reſolvend.

129 The Triple of 43.
5547 The triple Square of 43.

55599 The Diviſor.

729 The Cube of 9.
10449 The Square of 9 by 129.
49923 The triple Square by 9.

5097519 The Subtrahend.

....

259697989(638

216	
43697	Reſolvend.
18	The Triple of 6.
108	The triple Square of 6.
1098	The Diviſor.
27	The Cube of 3, the 2d Figure.
162	The triple Square of 3 by 18.
324	The triple Square 108 by 3.
34047	The Subtrahend.
9650989	Reſolvend.
189	The Triple of 63.
11907	The triple Square of 63.
119259	The Diviſor.
512	The Cube of 8.
12096	The Square of 8 by 189.
95256	The triple Square 11907 by 8.
9647072	The Subtrahend.
3917	The Remainder.

25917056(295.9
8

17917	The Reſolvend.
6	The Triple of 2.
12	The triple Square of 2.
126	The Diviſor.
729	The Cube of 9, the 2d Figure.
486	The Square of 9 by 6.
108	The triple Square by 9.
16389	The Subtrahend.
1528056	The Reſolvend.
87	The Triple of 29.
2523	The triple Square of 29.
25317	The Diviſor.
125	The Cube of 5, the 3d Fig.
2175	The Square of 5 by 87.
12615	The triple Square by 5.
1283375	The Subtrahend.
244681000	The Reſolvend.
885	The Triple of 295.
261075	The triple Square of 295.
2611635	The Diviſor.
729	The Cube of 9, the laſt Fig.
71685	The Square of 9 by 885.
2349675	The triple Square by 9.
235685079	The Subtrahend.
8995921	The Remainder.

In this Example I annex 3 Cyphers to the Remainder, which makes the 3d Reſolvend; by which means I bring one Place of Decimals. And ſo you may proceed to more decimal Places at Pleaſure, by annexing 3 Cyphers to the next Remainder, and carrying on the Work as before.

. . . .
22069810125(2805
8

14069 Reſolvend.

6 The Triple of 2.
12 The triple Square of 2.

126 The Diviſor.

512 The Cube of 8.
384 The Square of 8 by 6.
96 The triple Square by 8.

13952 The Subtrahend.
117810125 New Reſolvend.

84 The Triple of 28.
2352 The triple Square of 28.

23604 Diviſor.

840 The Triple of 280.
235200 The triple Square of 280.

2352840 New Diviſor.

125 Cube of 5.
21000 Square of 5 by 840.
1176000 Triple Square by 5.

117810125 Subtrahend.

. . . .

In this Example, 13952 being ſubtracted from the Reſolv. 14069, the Remainder is 117; to which bring down 810, the third Point, and it makes 117810, for a new Reſolv. and the next Diviſor is 23604, which you cannot have in the ſaid Reſolv. (the Unit's Place being rejected); ſo you muſt put 0 in the Quotient, and ſeek a new Diviſor (after you have brought down your laſt Point to the Reſolvend); which new Diviſor is 2352840; which you will find to be contained 5 times. So proceed to finiſh the reſt of the Work.

93759

. . . .
93759·575070(45·42
64

29759	The Reſolvend.
12	The Triple of 4, the firſt Figure.
48	The triple Square of 4.
492	The Diviſor.
125	The Cube of 5, the 2d Figure.
300	The Square of 5 by 12, the triple Quotient.
240	The triple Square by 5.
27125	The Subtrahend.
2634575	The Reſolvend.
135	The Triple of 45.
6075	The triple Square of 45.
60885	The Diviſor.
64	The Cube of 4.
2160	The Square of 4 by 135.
24300	The triple Square by 4.
2451664	The Subtrahend.
182911070	The Reſolvend.
1362	The Triple of 45.4.
618348	The triple Square of 45.4.
6184842	The Diviſor.
8	The Cube of 2.
5448	The Square of 2 by 1362.
236696	The triple Square of 2 by 2.
123724088	The Subtrahend.
59186982	The Remainder.

In extracting the Cube Root of a mix'd Number, always obſerve to make the decimal Part to conſiſt of either three, ſix, nine, *&c.* Places, that is, always to conſiſt of even Points, as in the laſt Example, where the decimal Places were five; to which I annex'd a Cypher to make up ſix, and ſo I proceed to point it; and by that means I have a Point falls upon the Unit's Place of whole Numbers, which you muſt always obſerve.

To extract the CUBE ROOT *out of a Fraction.*

This is the ſame to do as in whole Numbers, obſerve but the foregoing Directions for the true pointing thereof; for, as was before directed, the Decimal muſt always conſiſt of three, ſix, nine, *&c.* Places; and if it be not ſo, it muſt be made ſo, by annexing of Cyphers, as is above ſaid.

If the Cube Root of a vulgar Fraction be requir'd, you muſt firſt reduce it to a Decimal, and then extract the Root thereof.

Examples of each follow.

Example 1. Let the Cube Root of .401719179 be required.

$.40\dot{1}71\dot{9}17\dot{9}$(.737	Root.
343	
58719	Reſolvend.
21	Triple of 7.
147	Triple Square of 7.
1491	Diviſor.
27	Cube of 3.
189	Square of 3 by 21.
441	Triple Square by 3.
46017	Subtrahend.
12702179	Reſolvend.
219	Triple of 73.
15987	Triple Square of 73.
160089	Diviſor.
343	Cube of 7.
10731	Square of 7 by 219.
111909	Triple Square by 7.
11298553	Subtrahend.
1406236	Remainder.

Example 2. Let the Cube Root of .0001416 be requir'd.

.000141600(.052 Root.
125

16600 Reſolvend.

15 The Triple of 5.
75 Triple Square of 5.

765 Diviſor.

8 Cube of 2.
60 The Square of 2 by 15.
150 Triple Square by 2.

15608 Subtrahend.

992 Remainder.

Example 3. Let $\frac{2}{276}$ be a vulgar Fraction, whoſe Cube Root is requir'd.

By the firſt Rule of Chapter II. reduce the vulgar Fraction to a Decimal.

276)5.000000000(.018115942

2240
320
440
1640
2600
1160
560

8

.01811

. . .
.018115942(.262 Root.
8

10115	Reſolvend.
6	Triple of 2.
12	Triple Square of 2.
126	Diviſor.
216	Cube of 6.
216	Square of 6 by the Triple of 2.
72	The triple Square by 6.
9576	Subtrahend.
539942	Reſolvend.
78	Triple of 26.
2028	Triple Square of 26.
20358	Diviſor.
8	Cube of 2.
312	Square of 2 by 78.
4056	Triple Square 2028 by 2.
408728	Subtrahend.
131214	Remainder.

You may prove the Truth of the Work, by cubing the Root found, as was ſhew'd in the firſt Example; and if any thing remains, add it to the ſaid Cube, and the Sum will be the given Number, if the Work is rightly performed.

I will ſhew the Proof of the fifth Example, (*pag.* 50.) the given Number being 259697989, whoſe Root is 638, it being a ſurd Number, there remains 3917.

638
638

5104
1914
3828

The Square 407044
638

3256352
1221132
2442264

The Cube 259694072
The Remainder add 3917

Proof equal to the given Number 259697989

* * *

CHAP. IX.

Multiplication of Feet, Inches, and Parts.

In the multiplying of Feet, Inches, &c. I ſhall endeavour to lay down ſuch eaſy and familiar Rules, as may eaſily be underſtood by the meaneſt Capacity.

Example 1. Let 7 Feet 9 Inches be multiply'd by 3 Feet 6 Inches.

F.	I.	
7	9	
3	6	
23	3	*Pts.*
3	10	6
27	1	6

21
10 6
1 - 6
9
27:1:6

Firſt, Multiply 9 Inches by 3, ſaying, 3 times 9 is 27 Inches, which make 2 Feet 3 Inches; ſet down 3 under Inches, and carry 2 to the Feet, ſaying, 3 times 7 is 21, and 2 that I carry make 23; ſet down 23 under the Feet.

Then begin with 6 Inches, ſaying 6 times 9 is 54 Parts, which is 4 Inches and 6 Parts; ſet down 6 Parts, and carry 4, ſaying, 6 times 7 is 42, and 4 that I carry is 46 Inches, which is 3 Feet 10 Inches; which ſet down, and add all up together, and the Product is 27 Feet 1 Inch 6 Parts.

Example 2. Let 75 Feet 7 Inches be multiply'd by 9 Feet 8 Inches.

F.	I.	
75	7	
9	8	
680	3	
50	4	8
730	7	8

Firſt, Multiply by 9 Feet, ſaying, 9 times 7 is 63, which is 5 Feet 3 Inches; ſet down 3, and carry 5, ſaying, 9 times 5 is 45, and 5 I carry is 50; ſet down 0, and carry 5, ſaying 9 times 7 is 63, and 5 is 68; ſet down 68, and proceed to multiply by 8 Inches, ſaying 8 times 7 is 56; the Twelves in 56 are four times; and 8 remains: ſet 8 a Place to the Right-hand, and carry 4: Then multiply 75 by 8, and the Product is 600, and 4 that I carry is 604, which divided by 12, the Quotient is 50 Feet, and 4 remains; ſet down 50 Feet 4 Inches, and add all up together, and you will find the Product 730 Feet 7 Inches 8 Parts.

I will repeat the laſt Example again, and ſhew another Way to work it, which, I think, is better, and more expeditious, when then are more Figures than one in the Feet; thus,

F.	I.	
75	7	
9	8	
680	3	
25	2	4
25	2	4
730	7	8

Multiply by 9 Feet, firſt, as above directed; then, inſtead of multiplying by 8 Inches, let the 8 Inches be parted into ſuch aliquot or even Parts of a Foot, as you find to be contain'd in that Figure; if you take ſuch Parts of the Multiplicand, and add them to the former Product, the Sum will give the Anſwer: Thus, 8 Inches may be parted into 4 and 4, becauſe 4 is the third Part of 12. So, if you take the third Part of 75 Feet 7 Inches, and ſet it down twice, and add all together, the Sum will be 730 Feet 7 Inches 8 Parts, the ſame as before; thus, ſay how often 3 in 7, which is twice; ſet down 2; then, becauſe twice 3 is 6, ſay, 6 out of 7, and there remains 1, for which you muſt add 10 to the 5, and it makes 15; then the Threes in 15 are 5 times; ſet down 5; and, becauſe 3 times 5 is 15, there is 0 remains. Then go to the 7 Inches, ſaying, the Threes in 7 are twice; ſet down 2 in the Inches; and becauſe twice 3 is but 6, take 6 out of 7, and there remains 1 Inch, which is 12 Parts; then the Threes in 12 are 4 times, and 0 remains. So the third Part of 75 Feet 7 Inches, is 25 Feet 2 Inches 4 Parts; which ſet down again, and add all together, the Sum is 730 Feet 7 Inches 8 Parts; the ſame as before.

Example 3. Let 97 Feet 8 Inches be multiply'd by 8 Feet 9 Inches.

F.	I.
97	8
8	9
781	4
48	10
24	5
854	7

Begin, firſt, to multiply by 8 Feet, ſaying, 8 times 8 is 64 Inches, that is, 5 Feet 4 Inches; ſet down 4 Inches, and carry 5, ſaying, 8 times 7 is 56, and 5 I carry is 61; ſet down 1, and carry 6, ſaying, 8 times 9 is 72, and 6 I carry is 78, which ſet down: Then, inſtead of multiplying by 9 Inches, take the aliquot Parts of 12 which 9 makes, which is 6 and 3; 6 Inches being half 12, and 3 the fourth Part; therefore take the half of 97 Feet, 8 Inches, which is 48 Feet 10 Inches; and becauſe 3 is half 6, you may take the half of 48 Feet 10 Inches, which is 24 Feet 5 Inches; add all up together, and the Sum is 854 Feet 7 Inches. See the Work, as above,

Example 4. Let 75 Feet 9 Inches be multiply'd by 17 Feet 7 Inches.

F.	I.	P.
75	9	
17	7	
525		
75		
25	3	
18	11	3
8	6	
4	3	
1331	11	3

In this Example, becauſe there are more than 12 Feet in the Multiplier, therefore I firſt multiply the 75 by 17 Feet; then becauſe the aliquot Parts in 7 Inches are 4 and 3, that is, a third and a fourth, I take the third Part of 75 Feet 9 Inches, which is 25 Feet 3 Inches, and the fourth Part thereof is 18 Feet 11 Inches 3 Parts; then the aliquot Parts of 9 Inches are 6 and 3, that is, half and a fourth; therefore I take half 17 Feet, which is 8 Feet 6 Inches, and the fourth Part is 4 Feet 3 Inches (not meddling with the 7 Inches, becauſe that was multiply'd into the 9 before); then add all theſe together, and the Sum is 1331 Feet 11 Inches 3 Parts.

Example 5. Let 87 Feet 5 Inches be multiply'd by 35 Feet 8 Inches.

F.	I.	P.
87	5	
35	8	
435		
261		
29	1	8
29	1	8
11	8	0
2	11	0
3117	10	4

Work here as in the laſt Example. After you have multiply'd the Feet, then take the aliquot Parts of 8 Inches, which is two Thirds; therefore take the third Part of 87 Feet 5 Inches, and ſet it down twice. Thus the 3d Part of 87 Feet 5 Inches is 29 Feet 1 Inch 8 Parts; ſet this down twice: Then the aliquot Parts of 5 Inches are 4 and 1, that is, a 3d Part and a 12th Part; therefore take a third Part of 35, which is 11 Feet 8 Inches, and a 12th Part of 35 is 2 Feet 11 Inches; ſet all theſe one under another, and add them together, and the Sum is 3117 Feet 10 Inches 4 Parts.

Example 6. Let 259 Feet 2 Inches be multiply'd by 48 Feet 11 Inches.

F.	I.	P.
259	2	
49	11	
2972		
1036		
129	7	
86	4	8
21	7	2
8	0	0
12677	6	10

259 · 2
49 · 11
2331
1036
129 · 7
86 · 4 · 6
21 · 7 · 2
4 · 2 · 0
12936 · [illegible] · 10

Firſt, multiply the Feet; then take the aliquot Parts of 11, which will be 6, 4, and 1, that is, an half, a third, and a twelfth; therefore take the half of 259 Feet 2 Inches, which is 129 Feet 7 Inches, and a third Part is 86 Feet 4 Inches 8 Parts, and the twelfth Part of 259 Feet 2 Inches is 21 Feet 7 Inches 2 Parts; or (becauſe 1 is the fourth Part of 4.) you may more readily take the fourth Part of 86 Feet 4 Inches 8 Parts, which is alſo 21 Feet 7 Inches 2 Parts; then 2 Inches are the ſixth of 12, take the ſixth of 48 Inches will be 8 Inches, which place under the Inches; then add all together, and the Sum is 12677 Feet 6 Inches 10 Parts. See the foregoing Work.

I ſhall ſet down only the Working of ſome few Examples in Feet and Inches, and then proceed to multiply Feet, Inches, and Parts, *&c.*

	F.	I.	P.
	179	3	
	38	10	
	1432		
	537		
	89	7	6
	59	9	0
	9	6	0
Product	6960	10	6

	F.	I.	P.
	246	7	
	46	4	
	1476		
	984		
	82	2	4
	15	4	0
	11	6	0
Product	11425	0	4

	F.	I.	P.
	246	7	
	36	9	
	1476		
	738		
	123	3	6
	61	7	9
	12	0	0
	9	0	0
Product	9061	11	3

	F.	I.	P.
	257	9	
	39	11	
	2313		
	771		
	128	10	6
	85	11	0
	21	5	9
	19	6	0
	9	9	0
Product	10288	6	3

Example 11. Let 7 Feet 5 Inches 9 Parts be multiply'd by 3 Feet 5 Inches 3 Parts.

F.	I.	P.		
7	5	9		
3	5	3		
22	5	3	S	
3	1	4	9	T.
	1	10	5	3
25	8	6	2	3

In this Example, I firſt begin with 3 Feet, and thereby multiply 7 Feet 5 Inches 9 Parts: Firſt, I ſay, 3 times 9 is 27 Parts, that is, 2 Inches and 3 Parts; ſet down 3 under the Parts, and carry 2, ſaying, 3 times 5 is 15, and 2 I carry is 17, that is, 1 Foot 5 Inches; ſet down 5 Inches, and carry 1, and ſay, 3 times 7 is 21, and 1 I carry is 22; ſet down 22 Feet. Then begin with 5 Inches, ſaying, 5 times 9 is 45, which is 45 Seconds, which make 3 Parts and 9 Seconds; ſet down 9 Seconds a Place towards the Right-hand, and carry 3 Parts, ſaying, 5 times 5 is 25, and 3 I carry is 28, which is 2 Inches and 4 Parts; ſet down 4 Parts, and carry 2, ſaying, 5 times 7 is 35, and 2 I carry is 37, which is 3 Feet 1 Inch; ſet down 3 Feet 1 Inch, and begin to multiply by 3 Parts, ſaying, 3 times 9 is 27 Thirds, that is, 2 Seconds and 3 Thirds; ſet down 3 Thirds, and carry 2, ſaying, 3 times 5 is 15, and 2 I carry is 17, that is, 1 Part and 5 Seconds; ſet down 5 Seconds, and carry 1, ſaying, 3 times 7 is 21, and 1 I carry is 22, which is 1 Inch and 10 Parts, which ſet down, and add all up, and the Product is 25 Feet 8 Inches 6 Parts 2 Seconds 3 Thirds.

Note, That in multiplying Feet, Inches, and Parts, *&c.* if Feet be multiply'd by Feet, the Product is Feet; and Feet multiply'd by Inches, the Product is Inches;

and the twelfth Part is Feet; and Parts multiply'd by Feet, the Product is Parts, and the twelfth Part thereof is Inches; Parts multiply'd by Inches, the Product is Seconds, and the twelfth Part thereof is Parts; and Parts multiply'd by Parts, the Product is Thirds, and the twelfth Part thereof is Seconds. So that if you begin to multiply Parts by Feet in the firſt Row, and Parts by Inches in the ſecond Row, and Parts by Parts in the third Row, the firſt Figure in every Row will ſtand a Place more towards the Right-hand, as you may ſee in the laſt Example.

Example 12. Let 37 Feet 7 Inches 5 Parts be multiply'd by 4 Feet 8 Inches 6 Parts.

F.	I.	P.		
37	7	5		
4	8	6		
150	5	8	S	
12	6	5	8	
12	6	5	8	T
1	6	9	8	6
177	1	5	0	6

Firſt, I multiply by 4 Feet, ſaying 4 times 5 is 20, which is 1 Inch 8 Parts; ſet down 8, and carry 1, ſaying, 4 times 7 is 28, and 1 I carry is 29, which is 2 Feet 5 Inches; ſet down 5 Inches, and carry 2, ſaying, 4 times 7 is 28, and 2 I carry is 30; ſet down 0, and carry 3, and ſay, 4 times 3 is 12, and 3 is 15; ſet down 15. Then I begin with 8 Inches; but, becauſe the Feet in the Multiplicand are more than 12, it will be the beſt Way to work for the aliquot Parts of 8; ſo here I work for 4 Inches, and ſet that down twice, 4 being the third Part of 12; therefore take the third Part of 37 Feet 7 Inches 5 Parts, which is twelve Feet ſix Inches five Parts eight Seconds; ſet this down twice. Then begin with 6 Parts; but, inſtead of multiplying, take half 37 Feet 7 Inches 5

5 Parts, (becauſe 6 is half 12) and ſet it a Place more to the Right-hand: Thus, the half of 37 Feet is 18, which I muſt count 18 Inches, becauſe the Multiplier is 6 Parts; ſo the half of 37 Feet 7 Inches 5 Parts, is 1 Foot 6 Inches 9 Parts 8 Seconds 6 Thirds; which ſet down, and add all up together, and the Sum is 177 Feet 1 Inch 5 Parts 0 Seconds 6 Thirds.

Example 13. Let 311 Feet 4 Inches 7 Parts be multiply'd by 36 Feet 7 Inches 5 Parts.

F.	I.	P.	S	T
311	4	7		
36	7	5		
1866				
933				
103	9	6	4	
77	10	1	9	
8	7	9	6	4
2	1	11	4	7
12	0	0	0	0
1	0	0	0	0
	9	0	0	0
11402	2	4	11	11

In this Example, becauſe the Feet both in the Multiplier and Multiplicand are compound Numbers, I firſt multiply the Feet one by the other; then take the aliquot Parts of 7 Inches, which are 4 Inches and 3, that is, a third and a fourth Part; ſo take the third Part of 311 Feet 4 Inches 7 Parts, which is 103 F. 9 I. 6 P. 4 S. and the fourth Part is 77 F. 10 I. 1 P. 9 Sec. ſet theſe down one under another, the Feet under the other Feet; then the aliquot Parts of 5 Parts are 4 and 1, that is, a third and twelfth Part; ſo the third Part of 311 Feet 4 Inches 7 Parts is 103 F. 9 I. 6 P. 4 Sec. but becauſe the Multiplier is Parts, it muſt be ſet a Place to the Right-hand, that is, the 103 muſt be Inches, which is 8 Feet 7 Inches, therefore I ſet down 8 F. 7 I. 9 P. 6 Sec. 4 Th. Then, becauſe 1 Inch is a fourth Part or 4 Inches, therefore I take a fourth Part of 8 F. 7 I. 9 P. 6 Sec. 4 Th. which is 2 F. 1 I. 11 P. 4 Sec. 7 Th. which is the ſame as if I had taken a twelfth Part of 311 F. 4 I. 7 P. Then for 4 Inches in the Multiplicand, inſtead of multiplying 36 Feet by it, take a third Part, becauſe 4 Inches is the third Part of 12; ſo the third Part of 36 is 12 Feet, and the aliquot Parts of 7 Parts are 4 and 3, that is, a third and a fourth; ſo the third Part of 36 is 12, which now is 12 Inches, that is, 1 Foot, and the fourth Part is 9 Inches; add all theſe together, and the Sum will be 11402 Feet 2 Inches 4 Parts 11 Seconds 11 Thirds.

Example 14. Let 8 Feet 4 Inches 3 Parts 5 Seconds 6 Thirds be multiply'd by 3 Feet 3 Inches 7 Parts 8 Seconds 2 Thirds.

	F.	I.	P.	S.	T.				
	8	4	3	5	6				
	3	3	7	8	2				
	25	0	10	4	6				
	2	1	0	10	4	6			
		4	10	6	0	2	6		
			5	6	10	3	8	0	
				1	4	8	6	11	0
Prod.	27	7	3	5	1	8	8	11	0

In this laſt Example there is no Difficulty, if you do but obſerve the former Directions, and ſet every Row a Place more to the Right-hand.

I ſhall only ſet down the Working of ſome few Examples more, and ſo conclude this Chapter.

F.	I.	P.		
321	7	3		
9	3	6		
2894	5	3	S	
80	4	9	9	T
13	4	9	7	6
2988	2	10	4	6

F.	I.	P.	
42	7	8	
7	3	6	
298	5	8	
10	7	11	S
1	9	3	10
310	10	10	10

F.	I.	P.		
124	7	9		
14	6	2		
496				
124			S	
62	3	10	6	T
1	8	9	3	6
7	0	0	0	0
1	2	0	0	0
	7	0	0	0
	3	6	0	0
1809	[illegible]	1	9	6

1809·1·1·9·6

F.	I.	P.		
259	10	8		
18	5	4		
2072				
259			S	
86	7	6	8	
21	7	10	8	T
7	2	7	6	8
9	0	0	0	0
6	0	0	0	0
1	0	0	0	0
4793	6	0	10	8

F.	I.	P.		
267	7	10		
25	9	7		
1335				
534				
133	9	11	S	
66	10	11	6	
11	1	9	11	T
1	10	3	7	10
12	6	0	0	0
2	1	0	0	0
1	0	6	0	0
	8	4	0	0
6905	0	10	0	10

F.	I.	P.		
317	9	7		
37	5	9		
3219				
951			S	
105	11	2	4	
26	5	9	7	T
13	2	10	9	6
6	7	5	4	9
18	6	0	0	0
9	3	0	0	0
1	6	6	0	0
	3	1	0	0
11910	9	11	1	3

THE

THE
Complete Meaſurer, &c.

PART II.

CHAP. I.

The Menſuration of SUPERFICIES.

Superficial Figures are all ſuch as have only Length and Breadth, not having any commenſurable Thickneſs.

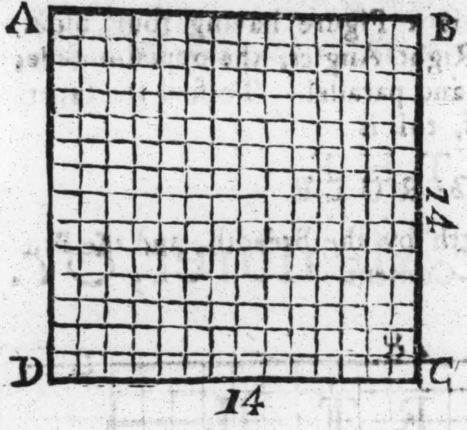
§ I. *Of a* SQUARE.

A SQUARE is a geometrical Figure, having four equal Sides, and as many right (or ſquare) Angles. To find the ſuperficial Content thereof, this is

The RULE.

Multiply the Side into itſelf, and the Product is the Content.

Let A B C D be a geometrical Square given, each Side being 14 Feet, Yards, Poles, or other Meaſures; multiply 14 by itſelf, and the Product is 196; which is the ſuperficial Content.



14
14
———
56
14
———
196 Product.

By Scale and Compaſſes.

Extend the Compaſſes from 1, in the Line of Numbers, to 14; the ſame Extent will reach from the ſame Point, turn'd forward to 196.

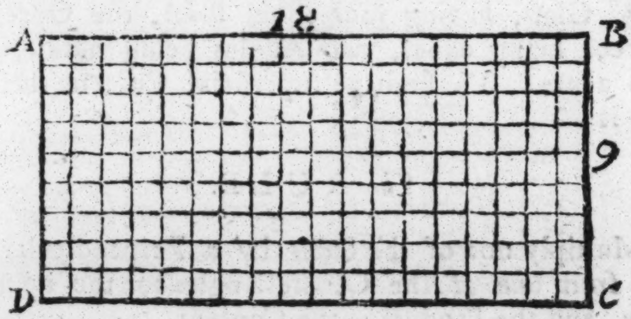
Demonſtration. Let each Side of the given Square be divided into 14 equal Parts, and Lines drawn from one another, croſſing each other within the Square; ſo ſhall the whole great Square be divided into 196 little Squares, as you may ſee in the Figure above, equal to the Number of Square Feet, Yards, or Poles, or other Meaſures, by which the Side was meaſur'd.

§ II. *Of a* PARALLELOGRAM, *or Long Square.*

A Parallelogram is a Figure having four Sides, and as many Right Angles, the oppoſite Sides thereof being equal and parallel. To find the ſuperficial Content thereof, this is

The RULE.

Multiply the Length by the Breadth, and the Product is the ſuperficial Content.



Length 18
Breadth 9
Product 162

Let A B C D be a long Square, the Length thereof 18 Feet, and the Breadth 9 Feet; which multiplied together, the Product is 162, the ſuperficial Content thereof.

By Scale and Compaſſes.

Extend the Compaſſes in the Line of Numbers from 1 to 9, the ſame Extent will reach from 18 down to 162 the Square Feet.

Demonſtration. If the Sides AB and CD be each divided into 18 equal Parts, repreſenting 18 Feet; and the Lines AD and BC each divided into 9 equal Parts, and Lines drawn from Point to Point, croſſing each other within the Figure; thoſe Lines will make thereby ſo many little Squares as there are Square Feet, *viz.* 162.

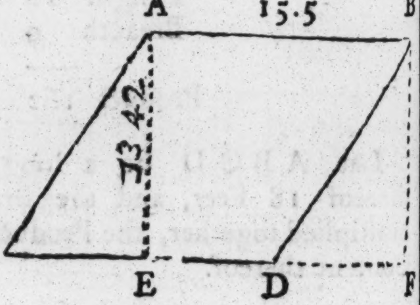
§ III. *Of a* RHOMBUS.

A Rhombus is a Figure repreſenting a Quarry of Glaſs, having four equal Sides, the Oppoſites thereof being equal, two Angles being obtuſe, and two acute. To find the ſuperficial Content thereof, this is

The RULE.

Multiply one of the Sides by a Perpendicular let fall from one of the Obtuſe Angles to the oppoſite Side, and the Product is the Content.

Perpend. 13 42
The Side 15.5
6710
6710
1342
Product 208.010



Let ABCD be a Rhombus given, whoſe Sides are each 15.5 Feet, and the Perpendicular EA is 13.42, which multiplied together, the Product is 208.010; which is the ſuperficial Content of the Rhombus, that is, 208 Feet and one hundredth Part of a Foot.

By

By Scale and Compaſſes.

Extend the Compaſſes from 1 to 13.42, that Extent will reach from 15.5, the ſame Way to 208 Feet the Content.

Demonſtration. Let CD be extended out to F, makeing DF equal to CE, and draw the Line BF; ſo ſhall the Triangle DBF be equal to the Triangle ACE: For DF and CE are equal and BF is equal to AE, becauſe AB and CF are parallel. Therefore the Parallelogram ABEF is equal to the Rhombus ABCD.

§ IV. *Of a* RHOMBOIDES.

A Rhomboides is a Figure having four Sides, the oppoſite whereof are equal and parallel; and alſo four Angles, the oppoſite whereof are equal. To find the ſuperficial Content thereof, this is

The RULE.

Multiply one of the longeſt Sides thereof by the Perpendicular let fall from one of the Obtuſe Angles to one of the longeſt Sides, and the Product is the Content.

```
  19.5
  10.2
 ------
   390
 195
 ------
198.90
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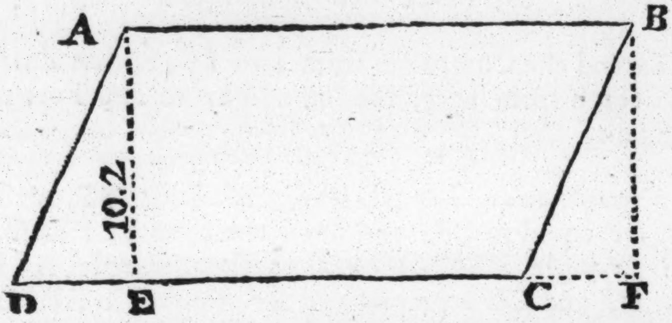


Let ABCD be a Rhomboides given, whoſe longeſt Sides, AB or CD is 19. 5 Feet, and the Perpendicular AE is 10. 2; which multiplied together, the Product is 198.9, that is, 198 ſuperficial Feet and 9 tenth Parts, the Content.

Demonſtration. If DC be extended to F, making CF equal to DE, and a Line be drawn from B to F; ſo will the Triangle CBF be equal to the Triangle ADE, and the Parallelogram AEFB be equal to the Rhomboides ABCD; which was to be proved.

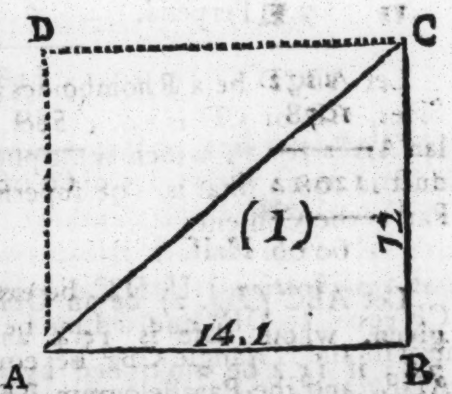
§ V. *Of a* TRIANGLE.

A Triangle is a Figure having three Sides and three Angles. Triangles are either right-angled or oblique-angled. Right-angled Triangles are ſuch as have one Right Angle. Oblique-angled Tringles are ſuch as have their Angles either acute or obtuſe. An Obtuſe Angle is greater than a Right Angle, that is, it is more than 90 Degrees; and an Acute Angle is leſs than a Right Angle. To find the ſuperficial Content thereof, this is

The

The RULE.

Let the Triangle be of what kind ſoever, multiply the Baſe by half the Perpendicular, or half the Baſe by the whole Perpendicular; or, multiply the whole Baſe by the whole Perpendicular, and take half the Product; any of theſe three Ways will give the Content.



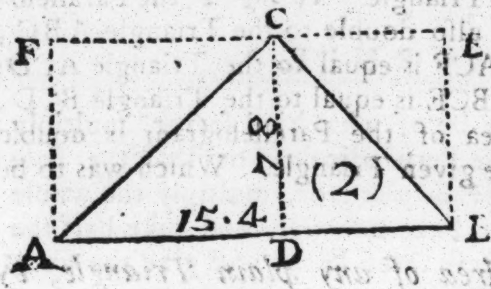
Let A B C be a Right-angled Triangle, whoſe Baſe is 14.1 Feet and the Perpendicular 12 F. Multiply 14.1 by 6, half the Perpendicular, and the Product is 84.6 Feet, the Content. Or, multiply 14.1 by 12, the Product is 169.2; the half thereof is 84.6, the ſame as before.

14.1 Baſe.
6 half Perpendicular.
84.6 Product

14.1 Baſe.
12 Perpendicular
169.2 Product.
84.6 Half.

By Scale and Compaſſes.

Extend the Compaſſes from 2 to 14.1, that Extent will reach the ſame Way from 12 to 84.6 Feet, the Content.



15.4 Baſe.
3.9 Half Perp.

1386
462

60 06 Product.

15.4 Baſe.
7.8 Perpend.

1232
1078

120.12

60.06 Half.

7.7 Half Baſe.
7.8 Perpendicular.

616.
539

60.06 Product.

Let ABC (*Fig.* 2) be an Oblique-angled Triangle given, whoſe Baſe is 15.4 and the Perpendicular 7.8; if 15.4 be multiplied by 3.9 (half the Perpendicular), the Product will be 60.06 for the Area or ſuperficial Content: Or, if the Perpendicular 7.8 be multiplied into half the Baſe 7.7, the Product will be 60.06 as before: Or, if 15.4, the Baſe, be multiplied by the whole Perpendicular 7.8 the Product will be 120.12, which is the double Area; the Half thereof is 60.06 Feet, as before. See the Work.

By Scale and Compaſſes.

Extend the Compaſſes from 2 to 15.4, that Extent will reach from 7.8 to 60 Feet, the Content.

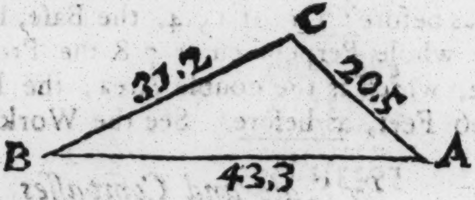
Demonſtration. If AD (*Fig.* 1.) be drawn parallel to BC, and DC parallel to AB; the Triangle ADC ſhall be equal to the given Triangle ABC. Hence the Parallelogram ABCD is double to the given Triangle; therefore half the Area of the Parallelogram is the Area of the Triangle. In *Fig.* 2. the Parallelogram ABEF is alſo double to the Triangle ABC; for the Triangle ACF is equal to the Triangle ACD, and the Triangle BCE is equal to the Triangle BCD; therefore the Area of the Parellelogram is double to the Area of the given Triangle. Which was to be proved.

To find the Area of any plain Triangle, by having the three Sides given, without the Help of a Perpendicular.

The RULE.

Add the three Sides together, and take half that Sum; then ſubtract each Side ſeverally from that half Sum. Which done, multiply that half Sum and the three Differences continually, and out of the laſt Product extract the Square Root; which Square Root ſhall be the Area of the Triangle ſought.

Example. Let ABC be a Triangle, whoſe three Sides are as followeth, *viz.* AB 43.3, AC 20.5, and BC 31.2, the Area is required.

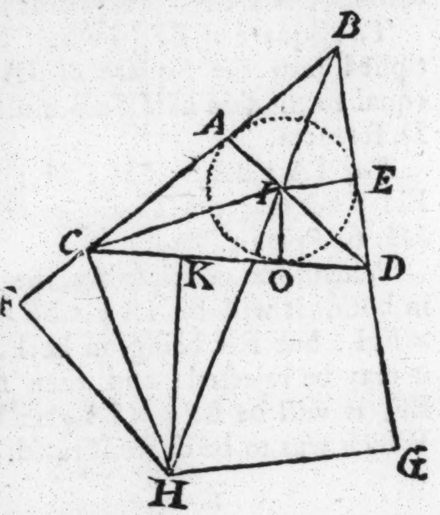


Sides	43.3	4.2	**Differences.**
	31.2	16.3	
	20.5	27.0	
Sum	95.0		
Half	47.5		

Area

```
Area 296.31        47.5  The half Sum.
                   27    Difference.
                 ------
                  3325
                  950
                 ------
                1282.5   Product.
                  16.3   Difference.
                 ------
                 38475
                76950
               12825
                 ------
             20904.75    Product.
                   4.2   Difference.
               --------
              4180950
             8361900
             ---------
              . .  .  .  .
             87799.9500(296.31
             4
             ---
          49)477
             441
             ----
         586)3699
             3516
             -----
        5923)18395
              17769
              ------
       59261)62600
              59261
              ------
               3339  Remains.
```

Demonſtration. In the Triangle BCD, I ſay, if from the half Sum of the Sides, you ſubtract each particular Side, and multiply the half Sum and the three Differences together, continually, the Square Root of the Product ſhall be the Area of the Triangle.



Firſt, by the Lines BI, CI, and DI, biſect the three Angles which Lines will all meet in the Point I; by which Lines the given Triangle is divided into three new Triangles, CBI, DCI, and BDI; the Perpendiculars of which new Triangles are the Lines AI, EI, and OI, being all equal to one another, becauſe the Point I is the Centre of the inſcribed Circle (by *Euclid, Lib.* 4. *Prop.* 4.): Wherefore to the Side BC join CF equal to DE, or DO; ſo ſhall BF be equal to half the Sum of the Sides, *viz.* $= \frac{1}{2} BC + \frac{1}{2} BD + \frac{1}{2} CD$.

And $BA = BF - CD$; for $CA = CO$ and $OD = CF$; therefore $CD = AF$; and $AC = BF - BD$, for $BE = BA$, and $ED = CF$; therefore $BD = BA + CF$, and $CF = BF - BC$.

Then make $CK = CF$, and draw the Perpendiculars FH, GH, and KH, and extend BI to H; becauſe the Angles $FCK + FHK$ are equal to two Right Angles (for the Angles F and K are Right Angles), equal alſo to $FCK + ACO$ (by *Euclid.* 1. 13.) and the Angles $ACO + AIO$ are equal to two Right Angles; therefore the Quadrangles FCKH and AIOC are alike; and the Triangles CFH and AIC are alſo ſimilar. And the Triangles BAI and BFH are likewiſe ſimilar.

From this Explanation, I ſay, the Square of the Area of the given Triangle will be $BFq \times IAq = BF \times BA \times CA \times CF$. In Words;

The Square of BF (the half Sum of the Sides) multiplied into the Square of IA ($=IF=IO$) will be equal to the ſaid half Sum multiplied into all the three Differences.

For $IA : BA :: FH : BF$; and $IA : CF :: AF : FH$; becauſe the Triangles are Similar. By *Euclid. Lib.* 6. *Prop.* 4.

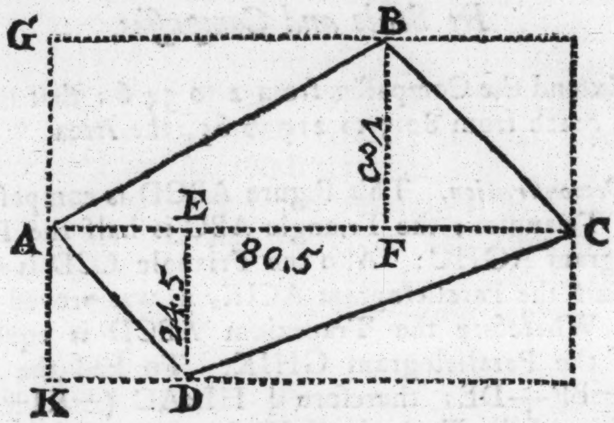
Wherefore multiplying the Extremes and Means in both, it will be $IAq \times BF \times FH = BA \times CA \times CF \times FH$; but FH being on both Sides of the Equation, it may be rejected; and then multiply each Part by BF, it will be $BFq \times IAq = BF \times BA \times CA \times CF$. Which was to be demonſtrated.

§ VI. *Of a* TRAPEZIUM.

A Trapezium is a Figure having four unequal Sides, and Oblique Angles. To find the Area or ſuperficial Content thereof, this is

The RULE.

Add the two Perpendiculars together, and take half the Sum, and multiply that half Sum by the Diagonal, or multiply the whole Sum by half the Diagonal, the Product is the Area. Or you may find the Area's of the two Triangles, ABC and ACD, (by Section V.) and add thoſe two Areas together, the Sum ſhall be the Area of the Trapezium.



BF=30.1
DE=24.5

Sum 54.6

Half 27.3
AC=80.5

1365
2184

Area 2197.65

Let ABCD be a Trapezium given, the Diagonal whereof is 89.5, and the Perpendicular BF 30.1, and the Perpendicular DE 24.5; theſe two added together, the Sum is 54.6, the Half thereof is 27.3, which multiplied by the Diagonal 80.5, the Product is 2197.65, which is the Area of the Trapezium; or if 40.25, half the Diagonal, be multiplied by 54.6, the whole Sum of the Perpendiculars, the Product is 2197.65, the ſame as before.

By

By Scale and Compaſſes.

Extend the Compaſſes from 2 to 54.6; that Extent will reach from 80.5 to 21976.65, the Area.

Demonſtration. This Figure ABCD is compoſed of two Triangles; the Triangle ABC is half the Parallelogram AGHC: Alſo the Triangle ACD is equal to half the Parallelogram ACIK, as was proved Sect. V. Wherefore the Trapezium ABCD is equal to half the Parallelogram GHIK. To find the Area HI=BF+DE; therefore $\frac{1}{2}$ HI×AC (=KI=GH) =Area of the Trapezium. Which was to be proved.

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§ VII. *Of Irregular* FIGURES.

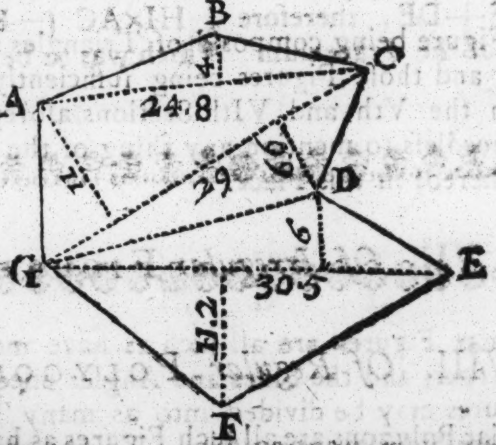
IRregular Figures are all ſuch as have more Sides than four, and the Sides and Angles unequal. All ſuch Figures may be divided into as many Triangles as there are Sides, wanting two. To find the Area of ſuch Figures, they muſt be divided into Trapeziums and Triangles, by Lines drawn from one Angle to another; and ſo find the Areas of all the Trapeziums and Triangles ſeverally, and then add all the Areas together; ſo will you have the Area of the whole Figure.

Let ABCDEFG be an irregular Figure given to be meaſured; firſt, draw the Lines AC and GD, and thereby divide the given Figure into two Trapeziums, ACGD and GDEF, and the Triangle ABC; of all which I find the Areas ſeverally.

Firſt, I multiply the Baſe AC by half the Perpendicular, and the Product is 49.6, the Area of the Triangle ABC.

Then for the Trapezium ACGD, the two Perpendiculars, 11 and 6.6, added together, make 17.6; the half thereof is 8.8, multiply'd by 29, the Diagonal; the Product is 255.2, the Area of that Trapezium.

And for the Trapezium GDEF, the two Perpendiculars, 11.2 and 6, added together, make 17.2; the Half thereof is 8.6; which multiply'd by 30.5, the Diagonal, the Product is 262.3, the Area thereof. All theſe Area's added together, make 567.1, and ſo much is the Area of the whole irregular Figure. See the Work.



24.8 Baſe AC.
2 Half Perpendicular.
———
49.6 Area of ABC.

11 Perpendiculars.
6.6
———
17.6 Sum.
8.8 Half.
29 Diag. CG.
———
792
176
———
255.2 Area of ACGD.

11.2 6	Perpendiculars.	30.5 8.6	
17.2	Sum.	1830 2440	
8.6	Half Sum.	262.30	Area of GDEF.
		255.2	Area of ACGD.
		49.6	Area of ABC.
		567.1	Sum of the Areas.

This Figure being compos'd of Triangles and Trapeziums, and thoſe Figures being ſufficiently demonſtrated in the Vth and VIth Sections aforegoing, it will be needleſs to mention any thing of the Demonſtration thereof in this Place.

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§ VIII. *Of Regular* POLYGONS.

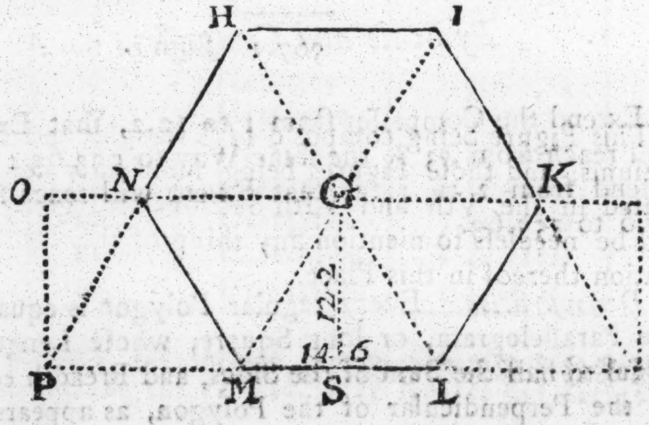
REgular Polygons are all ſuch Figures as have more than four Sides, all the Sides and Angles thereof being equal. Polygons are denominated from the Number of their Sides and Angles.

If the Figure conſiſts of		Equal Sides and Angles, it is called a regular	
	5		Pentagon.
	6		Hexagon.
	7		Heptagon.
	8		Octagon.
	9		Enneagon.
	10		Decagon.
	11		Endecagon.
	12		Dodecagon.

To find the Area or ſuperficial Content of any regular Polygon, this is

The RULE.

Multiply the whole Perimeter, or Sum of the Sides, by half the Perpendicular let fall from the Centre to the Middle of one of the Sides; or multiply the half Perimeter by the whole Perpendicular, and the Product is the Area.



14.6
3

43.8 Half Sum of the Sides.

12.64 The Perpendicular.
43.8 Half Sum.

10112
3792
5056

553.632 Area.

14.6
6

87.6 Sum of the Sides
6.32 Half Perpend.

1752
2628
5256

553 632 Area.

Let

Let HIKLMN be a regular Hexagon, each Side thereof being 14.6, the Sum of all the Sides is 87.6, the half Sum thereof is 43.8, which multiply'd by the Perpendicular GS 12 64, the Product is 553.63: Or, if 87.6, the whole Sum of the Sides, be multiply'd by half the Perpendicular 6.32, the Product is 553.63, the ſame as before, which is the Area of the given Hexagon.

By Scale and Compaſſes.

Extend the Compaſſes from 1 to 12.2, that Extent will reach from 43.8, the ſame Way to 553.63: Or, extend from 2 to 12.2, that Extent will reach from 87.6 to 553.63.

Demonſtration. Every regular Polygon is equal to the Parallelogram, or long Square, whoſe Length is equal to half the Sum of the Sides, and Breadth equal to the Perpendicular of the Polygon, as appears by the foregoing Figure; for the Hexagon HIKLMN is made up of ſix equilateral Triangles: And the Parallelogram OPQR is alſo compos'd of ſix equilateral Triangles, that is, five whole ones, and two Halves; therefore the Parallelogram is equal to the Hexagon.

A TABLE

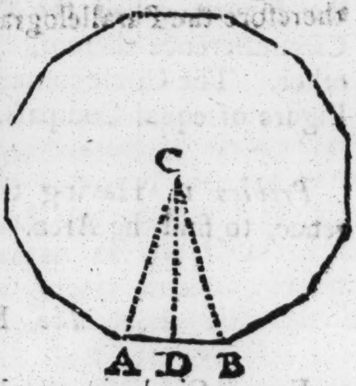
A TABLE *for the more ready finding the Area of a Polygon.*

Number of Sides.	Names.	Multipliers.
3	Trigon	.433013
4	Tetragon	1.000000
5	Pentagon	1.720477
6	Hexagon	2.598076
7	Heptagon	3.633959
8	Octagon	4.828427
9	Enneagon	6.181827
10	Decagon	7.694209
11	Endecagon	8.514250
12	Dodecagon	9.330125

Multiply the Square of the Side by the tabular Number, and the Product is the Area of the Polygon.

How to find theſe tabular Numbers.

Theſe Numbers are found by Trigonometry, thus: Find the Angle at the Centre of the Polygon by dividing 360 Degrees by the Number of Sides of the Polygon.



Example. Suppoſe each Side of the Dodecagon annexed by 1, and the Area be required.

Divide 360 by 12 (the Number of Sides), and the Quotient is 30 Degrees for the Angle ACB; the Half thereof is 15, the Angle DCB, whoſe Complement to 90 Degrees is 75 Degrees, the Angle CBD: Then ſay,

As s, DCB 15 Degrees is	Co-ar.	0.587004
to .5 the Half-ſide DB. Log.		1.698970
ſo is s, CBD 73 Degrees.		9.984944
to the Perpendicular CD 1.866025		0.270918

Then 1.866025 multiply'd by .5 (the Half-ſide), the Product is 9.330125 the Area of the Dodecagon requir'd.

§ IX. *Of a* CIRCLE.

A Circle is a plain Figure contained under one Line, which is called a Circumference, unto which all Lines drawn from a Point in the Middle of the Figure, called the Centre, and falling upon the Circumference thereof, are all equal the one to the other. The Circle contains more Space than any plain Figure of equal Compaſs.

Problem 1. Having the Diameter and Circumference, to find the Area.

The RULE.

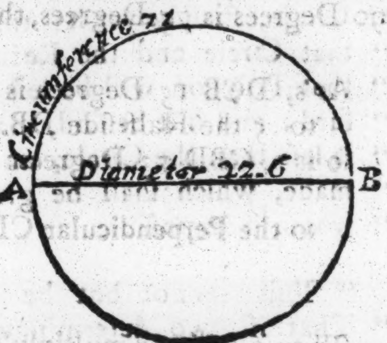
Every Circle is equal to a Parallelogram, whoſe Length is equal to half the Circumference, and the Breadth equal to half the Diameter; therefore multiply

ply half the Circumference by half the Diameter, and the Product is the Area of the Circle.

35.5 Half Circumf.
11.3 Half Diameter.

1065
355
355

401.15 Area.



Thus, if the Diameter of a Circle (that is, the Line drawn croſs the Circle through the Centre) be 22.6; and if the Circumference be 71, the Half of 71 is 35.5, and the Half of 22.6 is 11.3; which multiply'd together, the Product is 401.15, which is the Area of the Circle.

Demonſtration. Every Circle may be conceiv'd to be a Polygon of an infinite Number of Sides, and the Semidiameter muſt be equal to the Perpendicular of ſuch a Polygon, and the Circumference of the Circle equal to the Periphery of the Polygon; therefore half the Circumference, multiply'd by half the Diameter, gives the Area as aforeſaid.

Or (with *F. Ignat. Gaſton Pardies*), " Every Circle " is equal to a Rectangle-Triangle, one of whoſe " Legs is the Radius, and the other a right Line " equal to the Circumference of the Circle: For " ſuch a Triangle will be greater than any Polygon " inſcrib'd, and leſs than any Polygon circumſcrib'd " by the 24th, 25th, 26th, and 27th Articles of the " fourth Book of his Elements of Geometry); and " therefore muſt be equal to the Circle.

Diameter.

1.00000.00000.00000.00000.00000.00000.00000.

Circumference.

3.14159.26535.89793.23846.26433.83279.50288.

Of which large Number theſe ſix Places, 3.14159, anſwering to the Diameter 1.00000, may be ſufficient; of the three Proportions, as 7 to 22, 113 to 355, and 1 to 3.14159, I ſhall leave my Reader to uſe which of them he pleaſes, but ſhall commend the laſt two as moſt exact, tho' the firſt be moſt in Uſe; but in the following Work I ſhall uſe ſometimes one of them, and ſometimes another, but for the moſt Part that of *Van Culen*, as being moſt exact.

Problem 3. **Having the Circumference of a Circle, to find the Diameter.**

As 1 is to .318309, ſo is the Circumference to the Diameter.

Or, as 355 to 113, ſo is the Circumference to the Diameter.

Or, as 22 to 7, ſo is the Circumference to the Diameter.

Let the Circumference be 71, (as in the former Circle) if .318309 be multiply'd by 71, (as by the firſt Proportion) the Product will be 22.599939 for the Diameter. Or, by the ſecond Proportion, 113 multiply'd by 71, the Product is 8023; which divided by 355, the Quotient will be 22.6 the Diameter. Or, by the third Proportion, 71 multiply'd by 7, the Product is 497; this divided by 22, the Quotient is 22.5909, the Diameter.

.318309
71

318309
2228163

22.599939

113
71

113
791

355)8023(22.6
710

923
710

2130
2130

. . .

71
7

22)497(22.59
44

57
44

130
110

200
198

2

Thus, by both the firſt Proportions, the Diameter is 22.6, but by the laſt it falls ſomething ſhort.

By Scale and Compaſſes.

Extend the Compaſſes from 3.14159 to 1, that Extent will reach from 71 to 22.6, which is the Diameter ſought.

Or, you may extend from 1 to .318309.

Or from 22 to 7.

Or from 355 to 113; the ſame will reach from 71 to 22.6, as before.

Note, That if the Circumference be 1, the Diameter will be .318309.

Problem 4. Having the Diameter of a Circle, to find the Area.

All Circles are in Proportion one to another, as are the Squares of their Diameters (by *Euc.* 12.2). Now, the Area of a Circle, whoſe Diameter is 1, will be .785398, according to *Van Culen*'s Proportion before

As 14 : 11 : : 510.76
11

14)5618.36(401.31
56

18
14

43
42

16
14

2

Problem 5. Having the Circumference of a Circle to find the Area.

Becauſe the Diameters of Circles are proportional to their Circumferences; that is, as the Diameter of one Circle is to its Circumference; ſo is the Diameter of another Circle to its Circumference: Therefore the Areas of Circles are to one another, as the Squares of the Circumferences. And if the Circumference of a Circle be 1, the Area of that Circle will be .07958; then the Square of 1 is 1, and the Square of 71 (the Circumference of the former Circle) is 5041. Therefore it will be,

Sq. Cir. Area. Sq. Circumf.
As 1 : .07958 : : 5041
5041

7958
22 31832
4 397900

88 401.16278

Or thus:

As 88 : 7 : : 5041
7
88)35287(400.98 Area.
352
870
792
355   780
4   704
1420   76

Or, As 1420 : 113 : : 5041 : 401.15 Area.

Problem 6. By having the Diameter, to find the Side of a Square that is equal in Area to that Circle.

If the Diameter of a Circle be 1, the Side of a Square equal thereunto will be .8862. Therefore,

As 1 : .8862 : : 22.6 (the Diameter)
22.6
53172
17724
17724
To 20.02812 The Side of the Square AC.

As 14 : 11 : : 510.76
11

14)5618.36(401.31
56

18
14

43
42

16
14

2

Problem 5. Having the Circumference of a Circle. to find the Area.

Becauſe the Diameters of Circles are proportional to their Circumferences; that is, as the Diameter of one Circle is to its Circumference; ſo is the Diameter of another Circle to its Circumference: Therefore the Areas of Circles are to one another, as the Squares of the Circumferences. And if the Circumference of a Circle be 1, the Area of that Circle will be .07958; then the Square of 1 is 1, and the Square of 71 (the Circumference of the former Circle) is 5041. Therefore it will be,

Sq. Cir. Area. Sq. Circumf.
As 1 : .07958 : : 5041
5041

7958
22 31832
4 397900

88 401.16278

Cr

Or thus:

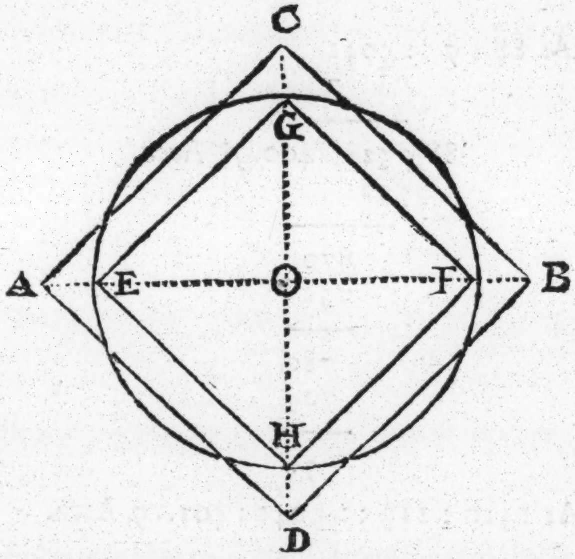
```
As 88 : 7 : : 5041
                 7
              ————
      88)35287(400.98 Area.
         352
         ————
           870
           792
           ———
355        780
  4        704
———        ———
1420        76
```

Or, As 1420 : 113 : : 5041 : 401.15 Area.

Problem 6. By having the Diameter, to find the Side of a Square that is equal in Area to that Circle.

If the Diameter of a Circle be 1, the Side of a Square equal thereunto will be .8862. Therefore,

```
As 1 : .8862 : : 22.6 (the Diameter)
           22.6
         ——————
          53172
         17724
        17724
        ———————
To 20.02812 The Side of the Square AC.
```



Let the Diameter of the Circle EF or GH, be 22.6 (as before), to find the Side of the Square AC, AD, *&c.* If .8862 be multiply'd by 22.6 the Product is 20.02812, which is the Side of a Square, equal in Area to the Circle given; for if 20.02812 be multiply'd Square-wiſe, that is, by itſelf, it will produce 401.1255907344, which is nearly equal to the Area found in the laſt Problem.

You may find the Side of the Square equal, by extracting the Square Root out of the Area of the given Circle.

```
       . . .
       401.15(20.0287295 Side of the Square.
       4
       ------
4002)01.1500
          8004
          --------
  40048)349600
   ....320384
          --------
          29216
          28034
          --------
           1182
            801
           ------
            381
            360
           ------
             21
             20
             --
              1
```

N. B. By this Method of extracting the ſquare Root of the Area, you may find the Side of a Square equal to any plain Figure, regular or irregular.

Problem 7. By having the Circumference, to find the Side of the Square equal.

If the Circumference of a Circle be 1, the Side of the Square equal will be .2821. Therefore,

```
        As 1 : .2821 : : 71 (the Circumference).
                      71
                    ------
                    2821
                  19747
                  --------
                 20.0291 The Side of the Square.
```

Problem 8. Having the Diameter, to find the Side of a Square, which may be inſcrib'd in that Circle.

If the Diameter of a Circle be 1, the Side of the Square inſcrib'd will be .7071. Therefore,

As 1 : .7071 : : 22.6.

22.6

42426
14142
14142

To 15.98046 The Side EG inſcrib'd.

Or, if you ſquare the Semidiameter, and double that Square, the ſquare Root of the doubled Square will be the Side of the Square inſcrib'd. For (by *Euclid.* 1.47) the Square of the Hypothenuſe EG is equal to the Sum of the other two Legs, EO and OG.

11.3 Semidiam.
11.3

339
113
113

127.69 The Squ. of EG, which double, becauſe (EO=OG.
2

· · ·
255.38(15.98 Root, which is the Side of the Sq.
1

25)155
125

309)3038
2781

3188)25700
25504

196

Problem 9. Having the Circumference, to find the Side of a Square which may be inſcrib'd.

If the Circumference be 1, the Side of the Square inſcrib'd will be .2251. Therefore,

As 1 : .2251 :: 71
71
2251
15757
15.9821 The Side of the Squ. EG.

Becauſe that in each of the four laſt Problems, *viz.* the 6th, 7th, 8th, and 9th, there is a Proportion laid down, it will be eaſy to work them with Scale and Compaſſes; for if you extend the Compaſſes from the firſt to the ſecond, that Extent will reach from the third to the fourth; as in the laſt Problem, where the Proportion is as 1 to .2251, ſo is 71 to the Side of the Square 15.9821. Here extend the Compaſſes from 1 to .2251; that Extent will reach from 71 to 15.98; and ſo of the reſt. But the fifth muſt be wrought like the 4th, thus; extend the Compaſſes from 1 to 71; that Extent turn'd over the ſame way from .07958, will fall, at the laſt, upon 401.15.

Problem 10. Having the Area, to find the Diameter.

If the Area of a Circle be 1, the Square of the Diameter thereof is 1.2732. Therefore,

Area. Sq. Diam. Area.
As 1 : 1.2732 :: 401.15.
401.15

63660
12732
12732
509280

. . . . . .
510.744180(22.599 The Diameter.
4

42)110
84

445)2674
2225

4509)44941
40581

45189)436080
406701

29379

By Scale and Compaſſes.

Extend the Compaſſes from 1 to 1.2732; that Extent will reach from 401.15 to 510.74, *&c*, Then divide the Space between 1 and 510.74 into two equal Parts, and you'll find the middle Point at 22.6. Or you may divide the Space upon the Line of Numbers, between 401.15 and .7854, into two equal Parts, and one of thoſe Parts will reach from 1 to 22.6, the Diameter ſought.

Problem 11. Having the Area, to find the Circumference.

If the Area of a Circle be 1, the Square of the Circumference will be 12.56637. Therefore,

Ar. Sq. Circumf. Area.
As 1 : 12.56637 : : 401.15
401.15

6283185
1256637
1256637
50265480

. . . . Circumference
5040.99932550(70.9990 Root.
49

1409)14099
12681

14189)141893
127701

141989)1419225
1277901

1419989)14132450
12779901

1352549

By Scale and Compaſſes.

Divide the Space between 401.15 and .07958, upon the Line, into two equal Parts; one of thoſe Parts will reach from 1 to 71, the Circumference ſought.

Problem 12. Having the Area, to find the Side of a Square inſcrib'd.

If the Area of a Circle be 1, the Area of a Square inſcrib'd within that Circle will be .6366. Therefore,

```
As 1 : 401.15 : : .6366
          .6366
         ───────
         240690
        240690
       120345
      240690
      ──────────
      .  .  .  .  .
      255.372090(15.98 Root, which is the Side of the
      1                                (Square ſought.
      ───
  25)155
      125
      ────
 309)3037
      2781
      ─────
3188)25620
      25504
      ─────
       11690
```

The ſame Reaſon may be given for the laſt Proportion, that was given before for the Proportion of Circles to the Squares of their Diameters and Circumferences; for not only the Squares of the Diameters and Circumferences are in Proportion to the Circles they belong to, but alſo all Figures inſcribed or circumſcrib'd, have the Squares of their like Sides proportional to the Circles they are inſcrib'd in, or circumſcrib'd about; and alſo to the Figures themſelves: The Square of any Side of one Figure is to the Area of that Figure, as the Square of the like Side of another ſimilar Figure is to the Area thereof: as you may find prov'd at large in *Euclid*, *Sturmius*, *Matheſis Enucleata*, and other Authors; but will be too large to inſert in this Place.

By Scale and Compaſſes.

Extend the Compaſſes from 1 to 401.15, that Extent will reach from .6366 to 255.37; the half Space between that and 1 is at 15.98, the Side of the Square.

Problem 13. Having the Side of a Square, to find the Diameter of the circumſcribing Circle.

If the Side of a Square be 1, the Diameter of a Circle that will circumſcribe that Square, will be 1.4142. Therefore,

As 1 : 1.4142 : : 15.98
15.98

113136
127278
70710
14142

22.598916 The Diameter ſought.

By Scale and Compaſſes.

Extend the Compaſſes from 1 to 1.4142, and that Extent will reach from 15.98 to 22.6, the Diameter ſought.

Problem 14. Having the Side of a Square, to find the Diameter of a Circle equal.

If the Side of a Square be 1, the Diameter of a Circle equal thereunto will be 1.128. Therefore,

Side Diam. Side of a Square.
As 1 : 1.128 : : 20.0291.
1.128

1602328
400582
200291
200291

22.5928248 Diameter.

By Scale and Compaſſes.

Extend the Compaſſes from 1 to 1.28; that Extent will reach from 20.0291, (the Side of the Squar given) to 22.6, the Diameter of the Circle ſought.

Problem 15. Having the Side of a Square, to find the Circumference of the circumſcribing Circle.

If the Side of a Square be 1, the Circumference o a Circle that will encompaſs that Square will b 4.443. Therefore,

Side Sq. Circum. Side Sq.
As 1 : 4.443 : : 15.98
15.98

35544
39987
22215
4443

70.99914 The Circumference.

By Scale and Compaſſes.

Extend the Compaſſe from 1 to 4.443, that Exte will reach from 15.98 to 71, the Circumference.

Probl

Problem 16. Having the Side of a Square, to find the Circumference of a Circle that will be equal thereunto.

If the Side of the Square be 1, the Circumference of a Circle that will be equal thereunto, ſhall be 3.545. Then,

```
As 1 : 3.545 : : 20.0291
                   3.545
                ---------
                 1001455
                 801164
               1001455
               600873
               ----------
               71.0031595 The Circumference.
```

By Scale and Compaſſes.

Extend the Compaſſes from 1 to 3.545, that Extent will reach from 20.0291 to 71, the Circumference ſought.

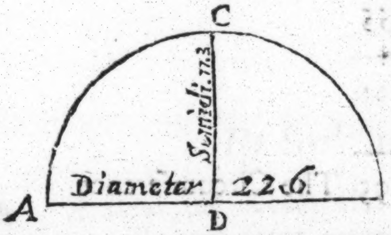
In ſeveral of the foregoing Problems, where the Diameter and Circumference is requir'd, the Anſwers are not exactly the ſame as the Diameter and Circumference of the given Circle, but are ſometimes too much, and ſometimes too little, as in the two laſt Problems, where the Anſwers in each ſhould be 71, the one being too much, and the other too little. The Reaſon of this is, the ſmall Defect that happens to be in the Decimal Fractions, they being ſometimes too great, and ſometimes too little; yet the Defect is ſo ſmall, that it is needleſs to calculate them to more Exactneſs.

§ X. *Of a* SEMICIRCLE.

TO find the Area of the Semicircle, this is

The RULE.

Multiply the fourth Part of the Circumference of the whole Circle (that is, half the Arch-line) by the Semidiameter, the Product is the Area.



Let ABC be a Semicircle, whoſe Diameter 22.6, and the half Circumference, or Arch-line, ACB, is 35.5. the Half thereof is 17.75, which multiply by the Semidiameter 11.3, and the Product is 200.575, the Area of the Semicircle.

17.75 The half Arch-line.
11.3 The Semidiameter.

5325
1775
1775

200.575 The Area of the Semicircle.

By Scale and Compaſſes.

Extend the Compaſſes from 1 to 11.3; that Extent will reach from 17.75 to 200.575, the Area.

If only the Diameter of the Semicircle be given, you may ſay, by the *Rule of Three*;

As 1 is to .3927, ſo is the Square of the Diameter to the Area.

By

By Scale and Compaſſes.

Extend the Compaſſes from 1 to the Diameter 22.6; that Extent turn'd twice from .3927, will reach, at the laſt, to 200.575.

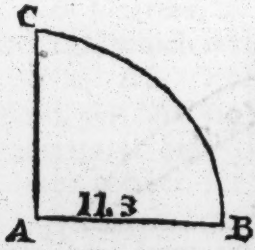
* * *

§ XI. *Of a* QUADRANT.

TO find the Area of a Quadrant, or fourth Part of a Circle, this is

The RULE.

Multiply half the Arch-line of the Quadrant (that is, the eighth Part of the Circumference of the whole Circle), by the Semidiameter, and the Product is the Area of the Quadrant.



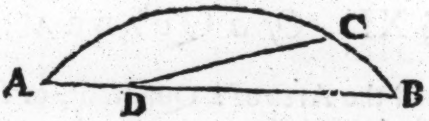
Let A B C be a Quadrant, or fourth Part of a Circle, whoſe Radius, or Semidiameter, is 11.3, and the half Arch-line 8.875; theſe multiply'd together, the Product is 100.2875 for the Area.

Theſe are the Rules and Ways commonly given for finding the Area of a Semicircle and Quadrant; but, I think, it is as good a Way, to find the Area of the whole Circle, and then take half that Area for the Semicircle, and a fourth Part for the Quadrant.

Before I proceed to ſhew how to find the Area of the Sector, and Segment of a Circle, I ſhall ſhew how to find the Length of the Arch-line, both *Geometrically* and *Arithmetically*.

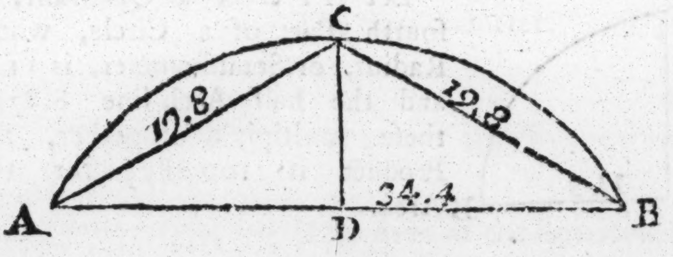
To find the Length of the Arch-line *Geometrically.*

Divide the Chord-line AB into four equal Parts, and ſet one of theſe Parts from B to C, and draw a Line from C to three of thoſe Parts at D; ſo ſhall CD be equal to half the Arch-line ACB.



To find the Length of the Arch-line *Arithmetically.*

Multiply the Chord of half the Segment AC or CB by 8, and from the Product ſubtract the Chord of the whole Segment AB, and divide the Remainder by 3; the Quotient is the Arch-line ACB ſought.



```
19.8 AC
   8
-----
158.4
 34.4 AB
-----
3)124
-----
Arch-line 41.333
```

Another

Another Way.

From the double Chord of half the Segment's Arch, ſubtract the Chord of the Segment, one third Part of the Difference added to the double Chord of half the Segment's Arch, the Sum is the Arch-line of the whole Segment.

Thus, if AC 19.8 be doubled, it makes 39.6; from which if you ſubtract 34.4, the Remainder is 5.2, which divided by 3, the Quotient is 1.733; this added to 39.6, (the double Chord of the half Segment) the Sum is 41.333. So if the Arch-line ACB was ſtretch'd out ſtrait, it would then contain 41.333 ſuch Parts as the Chord AB contains 34.4 of the like Parts.

Theſe two Rules may very eaſily be prov'd out of the Table of natural Sines; thus,

Suppoſe (in the former Figure) the Arch ACB to contain 120 Degrees; the natural Sine of half, *viz.* of 60 Degrees, is 86602, which being doubled, is 173204, which is the Chord of the whole 120 Degrees, that is, AB. Then, to find the Chord of the half Arch, *viz.* AC 60 Degrees, the Half of it 30 Degrees, the natural Sine thereof is 50000; which, doubled, makes 100000 for the Chord AC; then, according to the firſt Rule, multiply 100000 by 8, the Product is 800000; from which ſubtract 173204, (the Chord AB and the Remainder is 626796; which divide by 3, the Quotient is 208932, which is the Length of the Arch-line ACB, according to the firſt Rule. Now let us examine how near this comes to the true Quantity of the Arch propos'd. If the Radius or Semidiameter of a Circle be 100000, (as it is in the Table of Sines) then the Circumference will be 628318; and becauſe 120 Degrees is the third Part of the Circle, take the third Part of 628318 is 209439, which is the true Quantity of the Arch ACB in ſuch Parts as the Semidiameter contains 100000, and differs from that before found 507, which is a thing inconſiderable in *Practical Menſuration.* The latter of the foregoing Rules agrees exactly with the former, and therefore the Difference will be the ſame as above; either of the Rules gives the Quantity of the Arch-line too little, and the greater the Arch, the greater the Error. If you know the Degrees that are contain'd in the Segment's Arch, and would have the Arch-line very exactly, you may reaſon thus by the *Rule of Three:*

As the Circle's Periphery in Degrees, is to its Periphery in equal Parts; ſo is the Arch in Degrees and decimal Parts, to the ſame Arch in equal Parts.

Suppoſe the Circumference of a Circle be 71, and ſuppoſe the Arch to contain 52 Degrees, 15 Minutes, (the Decimal of 15 Minutes is .25) then ſay,

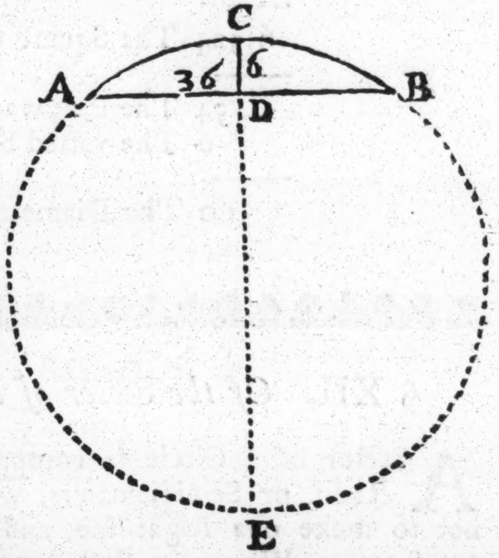
```
       Deg. Parts  Deg.
    As 360 : 71 :: 52.25
                     71
                  ------
                    5225
                  36575
                  ------
         36|0)370|9.75(10.305 fere.
              36
              ----
               109
               108
               ----
                 175
```

So the 52 Degrees 15 Minutes will contain 10.305 of ſuch Parts as the Circumference contains 71.

Thus I have ſhew'd ſeveral Ways of finding the Meaſure of the Curve-line of any Part of a Circle very near the Truth. The next thing I ſhall ſhew, is,

How

How to find the Diameter of a Circle by having the Chord and verſed Sine of the Segment Arithmetically.



Becauſe the Chord A B cuts the Diameter EC at right Angles, therefore the Semichord AD or D B is a mean proportional Line between the Parts of the Diameter C D and D E (by *Euc.* 6. 13); therefore, if you ſquare the Semichord AD or DB, and divide that Square, by the verſed Sine CD, the Quotient will be the Part of the Diameter wanting; to which add the given verſed Sine CD, and the Sum is the Diameter ſought.

Example. Let A C B be a Segment given, whoſe Chord A B is 36, and the verſed Sine CD 6; half 36 is 18, which ſquar'd, makes 324; this divided by 6, the Quotient is 54; to which add 6, the Sum is 60, the Diameter of the Circle CE.

18 Half the Chord.
18

144
18

6)324 The Square of AD.

54 The Part wanting DE.
6 The verſed Sine CD add.

60 The Diameter CE.

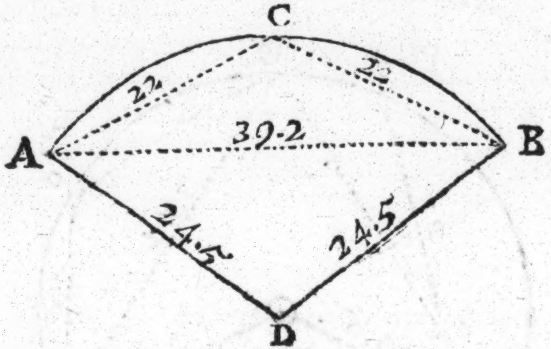
§ XII. *Of the Sector of a* CIRCLE.

A Sector of a Circle is comprehended under two Radii, or Semidiameters, which are ſuppos'd not to make one Right-line, and a Part of the Circumference: Whence a Sector may be either leſs or greater than a Semicircle. To find the Area or ſuperficial Content thereof, this is

The RULE.

Multiply half the Arch-line by the Semidiameter, and the Product is the Area.

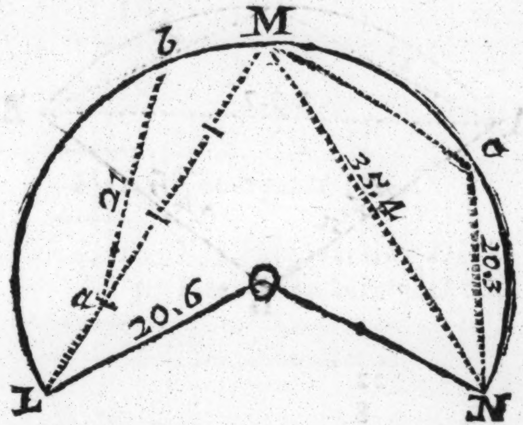
Let ADBC be the Sector of a Circle given, whoſe Semidiameter AD or BD is 24.5, and the Arch-line ACB (by the firſt Rule, *Pag.* 112.) I find to be 45.6; the Half thereof 22.8, being multiply'd by 24.5, (the Semidiameter) the Product is 558.6, which is the Area of the Sector ACBD.



22
8
———
176
39.2 Subtrahend.
———
3)136.8
———
45.6 Arch-line.
———
22.8 Half.
24.5 Semidiameter.
———
1140
912
456
———
558.60 The Area.

Again. Let LMNO be a Sector greater than a Semicircle, whoſe Semidiameter LO or NO is 20.6, and the Line *b a* equal to a fourth Part of the Arch-line L *b* MN 21, the Double whereof is 42, equal to the Arch-line L*b*M or M*c*N; or by the Arithmetical Rule, *Pag.* 112, the ſaid Arch is found to be 42.333, which multiply'd by 20.6, the Semidiameter, makes 872.0598 for the Area of the Sector LMNO.

See the following Work.



20.3 Chord N *c*
8
162.4
35.4 Chord MN.
3)12.70
42.333 Arch-line.

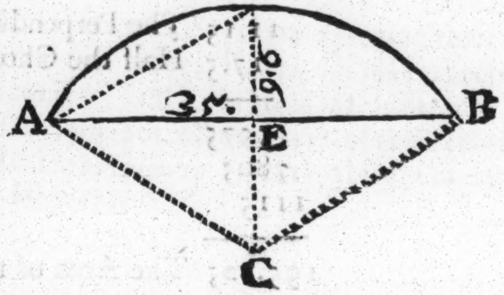
42.333
20.6
253998
84666
872.0598 Area.

§ XIII. *Of the Segment of a* CIRCLE.

A Segment of a Circle is a Part terminated by a Right-line leſs than the Diameter, call'd a Chord, and by a Part of the Circumference.

To find the Area of the Segment of a Circle, you muſt, firſt, find the Centre of the whole Circle, and draw the two Semidiameters, thereby completing the Sector, as in the following Figure. Then (by the laſt Section) find the Area of the whole Sector CADBC, and then (by Sect. 5.) find the Area of the Triangle ABC, and ſubtract the Area of the Triangle out of the Area of the Sector, the Remainder is the Area of the Segment.

Other-



Otherwiſe you may, without deſcribing the Figure, find the Semidiameter of the Circle by the Arithmetical Rule, *pag.* 113. and by the Arithmetical Rule, *pag.* 112. find the Arch-line; then multiply half the Arch-line by the Semidiameter; ſo have you the Area of the Sector. Then ſubtract the verſed Sine from the Semidiameter, the Remainder is the Perpendicular of the Triangle; and multiply the half Chord by the Perpendicular, the Product is the Area of the Triangle. Then ſubtract the Area of the Triangle from the Area of the Sector, and the Remainder is the Area of the Segment. See the Work.

```
    2)35=AB
     ------
       17.5
       17.5
     ------
       875
     1225
     175
    -------
9.6)306.25(31.9
   ------  9.6 Add.
     182   -----
     865   41.5 The Diameter of the Circle.
   ------  -----
        1  20.75 The Semidiameter.
            9.6  DE Subtrahend.
           ------
           11.15 Remains the Perpendicular EC.
```

11.15 The Perpendicular EC.
17.5 Half the Chord AE or EB.

5575
7805
1115

195.125 The Area of the Triangle.

306.25 The Square of AE.
92.16 The Square of DE the verſed Sine.

398.41 Sum.

The ſquare Root thereof is 19.96 The Chord AD.
8

159.68
Sub. 35 The Chord AB.

3)124.6

2)41.56 The Arch-line.

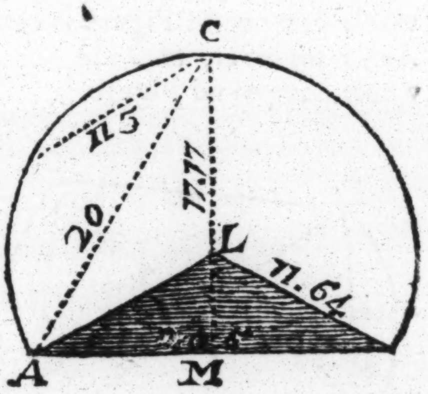
20.78 Half.
20.75 Semidiameter.

10390
14546
4156

From 431.1850 Area of the Sect.
Subtract 195.125 Ar. of the Trian.

Remains 236.060 Ar. of the Segm.

Again, let MACBM be a Segment greater than a Semicircle; obſerve the former Rules, in all reſpects as in the laſt Example; only, inſtead of ſubtracting the Area of the Triangle out of the Area of the Sector, here you muſt add it thereunto, as may plainly appear by the following Figure.



11.5
8

92.0
20

3)72

24 Half Arch-line.

11.64 Semidiam.
24

4656
2328

279.36 Area of the Sector LACBL.

17.17
11.64

5.53 LM.

10.25 Half the Baſe MA.
5.53 The Perpendicular LM.

3075
5125
5125

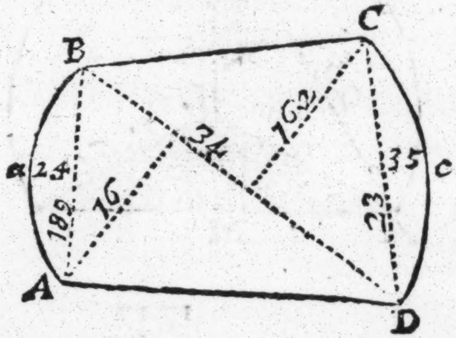
56.6825 The Area of the Triangle ALM
279.36 The Area of the Sector add.

336.0425 The Area of the Segment ſought.

§ XIV. *Of Compound* FIGURES.

MIX'D or compound Figures are ſuch as are compos'd of rectilineal and curvilineal Figures together.

To find the Area of ſuch mix'd Figures, you muſt find the Area of the ſeveral Figures of which the whole compound Figure is compoſed, and add all the Areas together, and the Sum will be the Area of the whole compound Figure.



16.2 } add.
16
32.2 Sum.
16.1 Half.
34 Diagonal.
644
483
547.4 Area of the (Trapezium,

10.236 Half the Arch-line A a B.
14.83 Semidiameter of the Arch AB.
30708
81888
40944
10236
151.79988 Area of the Sector.

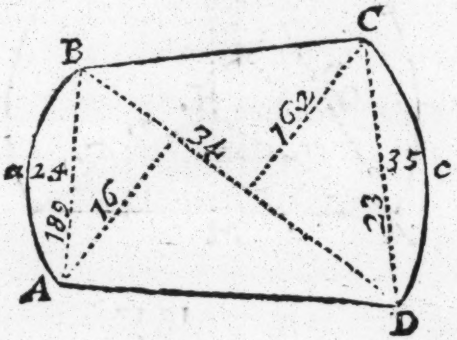
From

From	14.83	Semidiameter.
Subtract	3.4	Verſed Sine.
	11.43	Perpend. of the Triangle.
	9.45	Half the Chord AB.
	5715	
	4572	
	10287	
	108.0135	The Area of the Trian. ſubtracted from
	151.7999	The Area of the Sector.
	43.7864	The Area of the Segment A a BA.
	12.19	Half the Arch-line C c D.
	20.64	Semidiameter.
	4876	
	7314	
	24380	
	251.6016	The Area of the Sector.
From	20.64	The Semidiameter.
Subtract	3.5	Verſed Sine.
Remaind.	17.14	Perpendicular of the Triangle.
	11.5	Half the Chord DC.
	8570	
	1714	
	1714	
Sub.	197.110	Area of the Triangle.
From	251.602	Area of the Sector.
Rem.	54.492	The Area of the Segment Cc DC.
	43.786	The Area of the Segment Aa BA.
	547.4	The Area of the Trapezium.
Sum	645.678	The Area of the Whole.

§ XIV. *Of Compound* FIGURES.

MIX'D or compound Figures are ſuch as are compos'd of rectilineal and curvilineal Figures together.

To find the Area of ſuch mix'd Figures, you muſt find the Area of the ſeveral Figures of which the whole compound Figure is compoſed, and add all the Areas together, and the Sum will be the Area of the whole compound Figure.



16.2 } add.
16

32.2 Sum.

16.1 Half.
34 Diagonal.

644
483

547.4 Area of the (Trapezium,

10.236 Half the Arch-line A a B.
14.83 Semidiameter of the Arch AB.

30708
81888
40944
10236

151.79988 Area of the Sector.

From

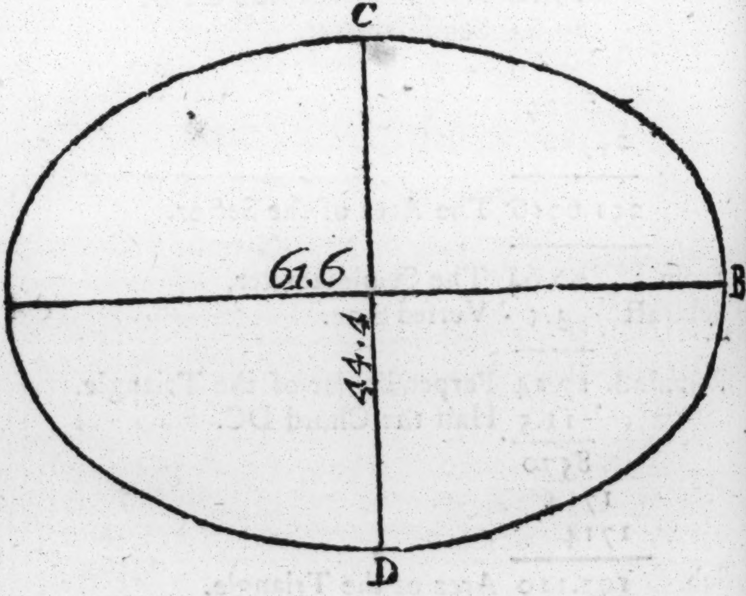
From	14.83	Semidiameter.
Subtract	3.4	Verſed Sine.
	11.43	Perpend. of the Triangle.
	9.45	Half the Chord AB.
	5715	
	4572	
	10287	
	108.0135	The Area of the Trian. ſubtracted from
	151.7999	The Area of the Sector.
	43.7864	The Area of the Segment A a BA.
	12.19	Half the Arch-line C c D.
	20.64	Semidiameter.
	4876	
	7314	
	24380	
	251.6016	The Area of the Sector.
From	20.64	The Semidiameter.
Subtract	3.5	Verſed Sine.
Remaind.	17.14	Perpendicular of the Triangle.
	11.5	Half the Chord DC.
	8570	
	1714	
	1714	
Sub.	197.110	Area of the Triangle.
From	251.602	Area of the Sector.
Rem.	54.492	The Area of the Segment Cc DC.
	43.786	The Area of the Segment Aa BA.
	547.4	The Area of the Trapezium.
Sum	645.678	The Area of the Whole.

§ XV. *Of an* ELLIPSIS.

AN Ellipſis, or Oval, is a Figure bounded by a regular curve Line, returning into itſelf; but of its two Diameters, cutting each other in the Centre, one is longer than the other, in which it differs from the Circle. To find the Area thereof, this is

The RULE.

Multiply the tranſverſe Diameter by the Conjugate, and multiply that Product by .7854, this laſt Product is the Area of the Oval.



61.6 The tranſverſe Diameter.
44.4 The conjugate Diameter.

2464
2464
2464

2735.04 The Rectangle.
.7854 The Area of Unity.

1094016
1367520
2188032
1914528

2148.100416 The Area of the Oval.

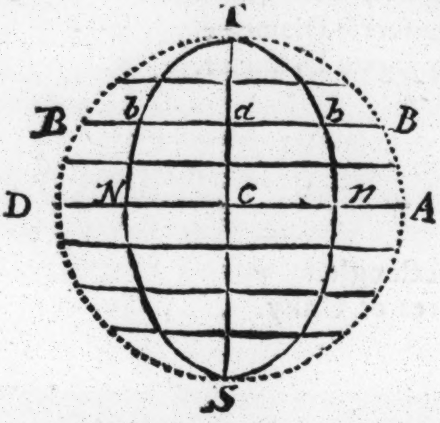
Demonſtration. If you circumſcribe any Ellipſis with a Circle, and ſuppoſe an infinite Number of Chord lines drawn therein, all parallel to the conjugate Diameter, as thoſe in the following Figure, then it will be,

As DA, the Diameter of the Circle, is to N n, the conjugate Diameter of the Ellipſis; ſo is B a B, any Chord in the Circle, to b a b, its reſpective Ordinate in the Ellipſis.

For, according to the Property of the Circle,

it is	1	a S × T a = □ B a.
And		by the Property of the Ellipſis.
it is	2	□ TC : □ NC : : a S × T a : □ b a.
1, 2,	3	□ TC : □ NC : : □ B a : □ b a.
3, hence	4	TC : NC : : B a : b a.
Conſeq.	5	TC : 2NC : : 2 B a : 2 b a.
That is	6	DA : Nn : : B a B : b a b.

But the Sum of an infinite Series of ſuch Chords as B a b, do conſtitute the Area of the Circle. And the Sum of the like Series of their reſpective Ordinates, as b a b, do conſtitute the Area of the Ellipſis:



Therefore, TS : Nn : : Circle's Area : the Ellipſis Area. But TS : Nn : : □ TS : TS × Nn; whence it follows, that,

□ TS : Circle's Area : : TS × Nn : Ellipſis Area.

Conſequently, As 1 is to .7854; ſo is the Rectangle, or Product of the tranſverſe and conjugate Diameter of any Ellipſis to its Area.

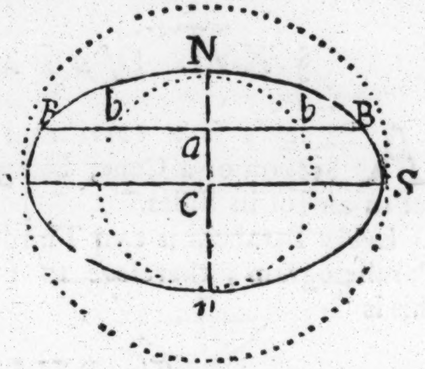
Hence it is eaſy to conceive, that the ſquare Root of the Product of the tranſverſe and conjugate Diameters will be the Diameter of a Circle equal to the Ellipſis.

Hence alſo the Segments of an Ellipſis, and its circumſcribing Circle, (whoſe Baſes are parallel to the conjugate Diameter, and of the ſame Height) are in Proportion one to another as their Baſes are; that is,

B a B : b a b : : Area Segment BTB : Area Segment b T B.

Or, TS : Nn : :Area Segment BTB : Area Segment b T b.

The Area of every Ellipſis is a mean Proportional between the Area of its circumſcribing and inſcrib'd Circles.



The Truth of this may eaſily be deduc'd from the laſt; for 'tis already prov'd, that □ TS : TS × Nn : : circumſcribing Circle's Area : Ellipſis Area.

But □ TS : T S × Nn : : T S × Nn : □ Nn. Therefore Ellipſis Area : inſcrib'd Circle's Area : TS× Nn : □ Nn.

Example. Let TS=36, and Nn=18.4. Then □ TS=1296, and □ Nn=338.56. Then 1296 × .7854=1017.8784 great Circle's Area; And 338.56×.7854=265.905,&c. leſſer Circle's Area; And 36 × 18.4=662.4 × .7854=520.24896, which is the Area of the Ellipſis; then it will be,

1017.878 : 520.24896 : : 520.24896 : 265.905024.

That is, As the great Circle's Area is to the Area of the Ellipſis; ſo is the Area of the Ellipſis to the Area of the leſſer Circle.

From hence it follows, that all Segments of an Ellipſis, and its inſcrib'd Circle, (whoſe Baſes are parallel to the tranſverſe Diameter, and have the ſame Height) are in Proportion one to another as the Area of the Ellipſis and Circle are.

That is, As the Area of the Circle is to the Area of the Ellipſis, ſo is the Segment b N b : to the Segment BNB,

Or, Nn : TS : : Area Segment b N b : Area Segment BNB.

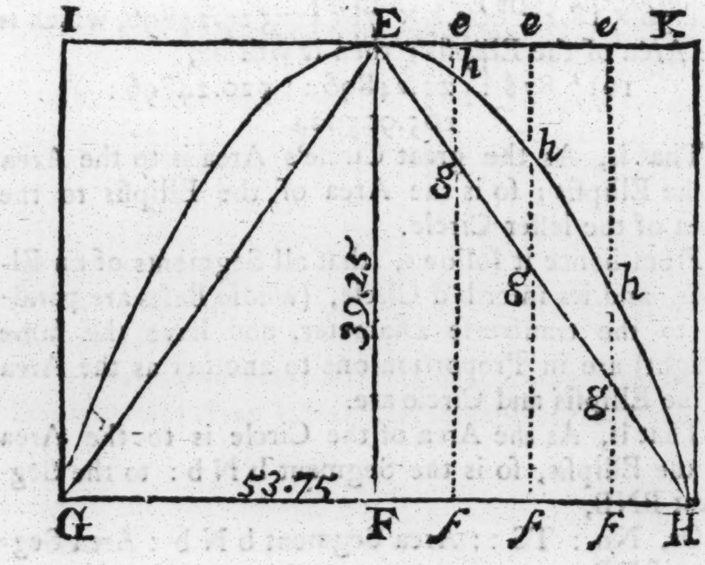
§ XVI. *Of a* PARABOLA.

A Parabola is a curvilineal Figure, made by the Section of a Cone, being cut by a Plane parallel to one of its Sides.

Every Parabola is two Thirds of its circumſcribing Parallelogram; therefore to find the Area thereof, this is

The RULE.

Multiply the Baſe, or greateſt Ordinate, by the perpendicular Height, and multiply that Product by 2, and divide the laſt Product by 3, the Quotient will be the Area of the Parabola.



53.75 The Ordinate GH.
39.25 The Perpendicular EF.

26875
10750
48375
16125

2109 6875
2

3)4219.3750

1406.4583 The Area.

Demonſtration. Let FH, the Semi-ordinate, be divided into four equal Parts, or into 8.16, *&c.* and through the Diviſions draw Lines, as e f, ef, *&c.* parallel to the Axis EF. Suppoſe alſo EF to be 4.

Then, I ſay, the Parabolic Sphere Eh HF is to the Parallelogram EKFH as 2 to 3; but to the Triangle EFH as 4 to 3.

For, firſt, gf, gf, gf, *&c.* are in continual arithmetical Proportion from the Nature of plain Triangles.

Secondly, f e : g e : : g e : h e; but h e in the Axis EF=o. and in the firſt Parallel e f muſt be equal to $\frac{1}{4}$, in the next e f muſt be equal to $\frac{4}{4}$, in the third to $\frac{9}{4}$, and ſo on, in a duplicate arithmetical Progreſſion.

For e f (=4) : g e (=1) : : g e (=1) : e h (=$\frac{1}{4}$). And the ſecond e f (=4) : e g (=2) : : e g (=2) : e h (=$\frac{4}{4}$), *&c.* And thus it will be, if the Lines F f, f f, *&c.* be again biſected, *&c. ad infinitum,* ſo that all the Indiviſibles of the trilinear Space E K H h E will be in a duplicate arithmetical Progreſſion increaſing. But the Sum of a Rank of ſuch Terms is ſubtriple to a Rank of as many equal to the greateſt (by Lemma 3); wherefore the whole trilinear Space EKHhE is to the Parallelogram as 1 to 3; and, conſequently, the remaining parabolic Space muſt be to it as 2 to 3; which was to be proved.

And ſince the Triangle FEH is to the Parallelogram as 1 to 2, it muſt be to the Parabola as $1\frac{1}{2}$ to 2, or as 3 to 4; which was to be prov'd.

Before I proceed to the Menſuration of ſolid Bodies, I will lay down ſuch Lemmas as will be neceſſary to facilitate the Demonſtration of all ſuch Solids.

LEMMA I.

In any Series of equal Numbers (repreſenting Lines or other Quantities) as 1, 1, 1, 1, *&c.* or 2, 2, 2, 2, *&c.* or 3, 3, 3, 3, *&c.* if one of the Terms be multiply'd into the Number of Terms, the Product will be the Sum of all the Terms in the Series.

LEMMA II.

If a Series of Numbers, in arithmetical Progreſſions, begin with a Cypher, and the common Difference be 1, as 0, 1, 2, 3, *&c.* (repreſenting a Series of Lines or Roots beginning with a Point) if the laſt Term be multiply'd into the Number of Terms, the Product will be double the Sum of all the Series.

That is, putting L = the laſt Term, N = the Number of Terms, and S = the Sum of all the Series; then will $NL = 2S$; conſequently, $\frac{1}{2}NL = S$, *viz.* One-half of ſo many times the greateſt Term as there are Numbers of Terms in the Series.

$$\text{Thus}\left\{\begin{array}{l} \underline{0+1+2+3+4=10} \text{ the Sum} = \frac{1}{2}NL. \\ 4+4+4+4+4=20=NL. \end{array}\right.$$

LEMMA III.

If a Series of Squares, whoſe Sides or Roots are in arithmetical Progreſſion, beginning with a Cypher, *&c.* be infinitely continued, the laſt Term, being multiply'd into the Number of Terms, will be triple to the Sum of all the Series, *viz.* NLL$=$3S; or, $\frac{1}{3}$ NLL$=$S.

That is, the Sum of ſuch a Series will be One-third of the laſt or greateſt Term, ſo many times repeated as there are Numbers of Terms in their Series.

Inſtances in ſquare Numbers.

$$1\ \left\{\frac{0+1+4}{4+4+4}=\frac{5}{12}=\frac{1}{3}+\frac{1}{12}.\right.$$

$$2\ \left\{\frac{0+1+4+9}{9+9+9+9}=\frac{14}{56}=\frac{7}{18}=\frac{1}{5}+\frac{1}{18}.\right.$$

$$3\ \left\{\frac{0+1+4+9+16}{16+16+16+16+16}=\frac{30}{80}=\frac{3}{8}=\frac{9}{24}=\frac{1}{3}+\frac{1}{13}.\right.$$

(&c.

From theſe Inſtances it is evident, that as the Number of Terms in the Series do increaſe, the Fraction or Exceſs above $\frac{1}{3}$ does increaſe, the ſaid Exceſs always being $\frac{1}{6N-6}$; which, if we ſuppoſe the Series to be infinitely continued, will quite vaniſh, and become nothing at all.

LEMMA IV.

If a Series of Cubes, whoſe Roots are in arithmetical Progreſſion, beginning with a Cypher, &c. (as above) be infinitely continued, the Sum of all the Series will be $\frac{1}{4}$ NLLL$=$S.

That is, One-fourth of the laſt Term ſo many times repeated as there are Numbers of Terms.

Inſtances in cube Numbers.

If 0, 1, 2, 3, 4, 5, *&c.* be the Roots of the Cubes,

1 $\left\{\frac{0+1+8+27}{27+27+27+27}\right.=\frac{36}{108}=\frac{4}{12}=\frac{1}{4}+\frac{1}{12}.$

2 $\left\{\frac{0+1+8+27+64}{64+64+64+64+64}\right.=\frac{100}{320}=\frac{10}{32}=\frac{5}{16}=\frac{1}{4}+\frac{1}{16}$

3 $\left\{\frac{0+1+8+27+64+125}{125+125+125+125+125+125}\right.=\frac{225}{750}=\frac{45}{150}.$

$(=\frac{6}{20}=\frac{1}{4}+\frac{1}{20}.$

From theſe Examples it plainly appears, that as the Number of Terms in the Series increaſes, the Fraction or Exceſs above $\frac{1}{4}$ decreaſes, the Exceſs being always $\frac{1}{4N-4}$; which, if we ſuppoſe the Series to be infinitely continued, will become infinitely ſmall, or nothing.

LEMMA V.

If a Series of Biquadrates, whoſe Roots are in arithmetical Progreſſion, beginning with a Cypher, *&c.* as before, be infinitely continued, the Sum of all the Terms in ſuch a Series will be $\frac{1}{5}$ NLLLL.

The Truth of this may be manifeſt by the like Proceſs as in the foregoing Lemma's, and ſo on for higher Powers.

LEMMA VI.

The Sum of an infinite Progreſſion, whoſe greateſt Term is a ſquare Number, the other decreaſing by odd Numbers, *viz.* 1, 3, 5, *&c.* is in ſubſeſquialteran Proportion of the Sum of the like Number of equal Terms, that is, as 2 to 3.

Inſtances in ſuch Progreſſions.

1 $$\left\{\frac{9+8+5}{9+9+9}\right. = \frac{22}{27} = \frac{2}{3} + \frac{4}{9}.$$

2 $$\left\{\frac{16+15+12+7}{16+16+16+16}\right. = \frac{50}{64} = \frac{2}{3} + \frac{15}{32}.$$

3 $$\left\{\frac{25+24+21+16+9}{25+25+25+25+25}\right. = \frac{95}{125} = \frac{2}{3} + \frac{7}{27}.$$

4 $$\left\{\frac{36+35+32+27+20+11}{36+36+36+36+36+36}\right. = \frac{161}{216} = \frac{2}{3} + \frac{17}{72}.$$

From theſe Examples it plainly appears, that, as the Number of Terms in the Series increaſes, the Fraction or Exceſs above $\frac{2}{3}$ decreaſes; and if we ſuppoſe the Series to be infinitely continued, that Exceſs will quite vaniſh, and the Sum of the infinite Series will be $\frac{2}{3}$ of ſo many equal to the greateſt.

CHAP. II.

The Menſuration of SOLIDS.

SOLID Bodies are ſuch as do conſiſt of Length, Breadth, and Thickneſs; as Stone, Timber, Globes, Bullets, *&c.*

§ I. *Of a* CUBE.

A CUBE is a ſquare Solid, comprehended under ſix geometrical Squares, being in the Form of a Dye. To find the ſolid Content, this is

The RULE.

Multiply the Side of the Cube into itſelf, and that Product again by the Side; the laſt Product will be the Solidity, or ſolid Content of the Cube.

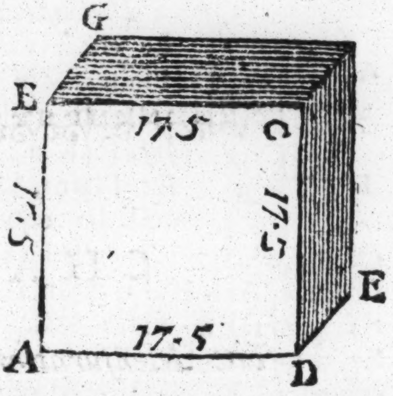
17.5
17.5

875
1225
175

306.25
17.5

153125
214375
30625

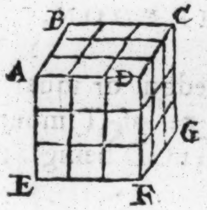
5359 375 The ſolid Content of the Cube.



Suppoſe ABCDEFG a cubical Piece of Stone or Wood, each Side thereof being 17 Inches and an half; multiply 17.5 by 17.5, and the Product is 306.25; which being multiply'd by 17.5, the laſt Product is 5359.375, which is 5359 ſolid Inches and 375 Parts. To reduce the ſolid Inches to Feet, divide by 1728, (becauſe ſo many cubical Inches is a Foot) and the ſolid Feet in the Cube will be 3, and 175 cubical Inches remain.

By Scale and Compaſſes.

Extend the Compaſſes from 1 to 17.5; that Extent turn'd over twice from 17.5 will reach to 5359, the ſolid Content in Inches. Then extend the Compaſſes from 1728 to 1; that Extent, turn'd the ſame way from 5359, will reach to 3.1 Feet.

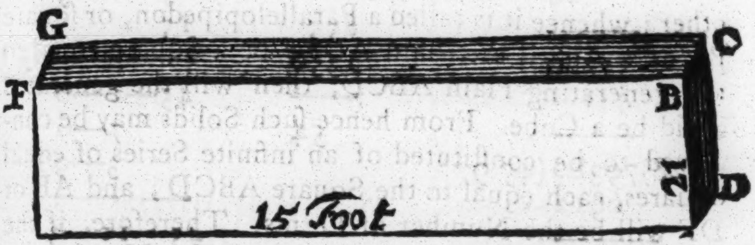


Demonſtration. If the Square ABCD be conceiv'd to be mov'd down the Plain ADEF, always remaining parallel to itſelf, there will be generated, by ſuch a Motion, a Solid, having ſix Plains, the two oppoſite whereof will be equal and parallel to each other; whence it is called a Parallelopipedon, or ſquare Priſm. And if the Plain ADEF be a Square equal to the generating Plain ABCD, then will the generated Solid be a Cube. From hence ſuch Solids may be conceived to be conſtituted of an infinite Series of equal Squares, each equal to the Square ABCD; and AE or DF will be the Number of Terms. Therefore, if the Area of ABCD be multiply'd into the Number of Terms AE, the Product is the Sum of all the Series, (*per* Lemma I.) and, conſequently, the Solidity of the Parallelopipedon or Cube. Or, if the Baſe ABCD, being divided into little ſquare Areas, be multiply'd into the Height AE, divided by a like Meaſure for Length, after this Way you may conceive as many little Cubes to be generated in the whole Solid, as is the Number of the little Areas of the Baſe multiply'd by the Number of Diviſions the Side AE contains. Thus, if the Side of the Baſe AB be 3, that multiply'd into itſelf is 9, which is the Area of the ſquare Baſe ABCD; then, if AE be likewiſe 3, multiply 9 by 3, and the Product is 27; and ſo many little Cubes will this Solid be cut into, if you conceive it to be cut as the Lines direct.

From this Demonſtration it is very plain, that, if you multiply the Area of the Baſe of any Parallelopipedon into its Length or Height, that Product will be the ſolid Content of ſuch a Solid,

§ II. *Of a* PARALLELOPIPEDON.

LET ABCDEFG be a Parallelopipedon, or ſquare Priſm, repreſenting a ſquare Piece of Timber or Stone, each Side of its ſquare Baſe ABCD being 21 Inches, and its Length AE 15 Feet.



Firſt, then, multiply 21 by 21, the Product is 441, the Area of the Baſe in Inches; which multiply'd by 180, the Length in Inches, and the Product is 79380, the ſolid Content in Inches. Divide the laſt Product by 1728, and the Quotient is 45.9, that is, 45 ſolid Feet and 9 Tenths of a Foot. Or thus: Multiply 441 by 15 Feet, and the Product is 6615; divide this by 144, and the Quotient is 45.9, the ſame as beſore.

Or thus, by multiplying Feet and Inches.

Multiply 1 Foot 9 Inches by 1 Foot 9 Inches, and the Product is 3 Feet 0 Inches 9 Parts; this multiply'd again by 15 Feet, gives 45 Feet 11 Inches 3 Parts, that is, 45 Feet and $\frac{11}{12}$ of a Foot and $\frac{1}{4}$ of $\frac{1}{12}$.

See the Work of all theſe.

21
21
21
42
441
180
35280
441
1728)79380(45.9
6912
10260
8640
16200
15552
648

441
15
2205
441
144)6615(45.9
855
1350
54

F. I.
1—9
1—9
1—9
1—3—9
3—0—9
15
45—0—0
7—6
3—9
45—11—3

By Scales and Compaſſes.

Extend the Compaſſes from 12 to 21, and that Extent will reach to near 46 Feet, being twice turn'd over from 15 Feet; ſo the ſolid Content is almoſt 46 Feet.

If the Baſe of the ſquar'd Solid be not an exact Square, but in Form of a rectangle Parallelogram, the Way of meaſuring it is much the ſame; for, firſt, you muſt find the Area of the Baſe by multiplying the Breadth by the Depth; and then multiply that Area by the Length of the Piece, as before: Thus, If a Piece of Timber be 25 Inches broad, 9 Inches deep, and 25 Feet long, how many ſolid Feet are contained therein?

25	F. I.
9	2—1
	0—9
225	
25	1—6—9
	25
1125	
450	25—0—0
	12—6—0
144)5625(39	1—0—6
432	0—6—3
1305	39—0—9
1296	
9	Anſwer 39 Feet.

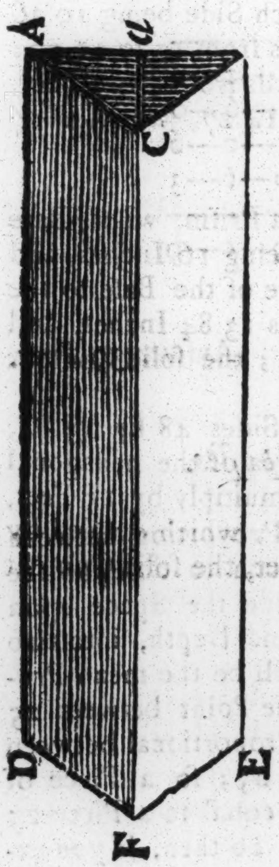
By Scale and Compaſſes.

Firſt, find a mean geometrical Proportion between the Breadth and the Depth; which to do upon the Line of Numbers, you muſt divide the Space upon the Line, between the Breadth and Depth, into two equal Parts; that middle Point will be the mean Proportional ſought: Thus the middle Point between 25 and 9 is at 15; ſo is 15 a mean Proportional between 9 and 25, for, 9 : 15 :: 15 : 25; ſo a Piece of Timber of 15 Inches ſquare is equal to a Piece 25 Inches broad and 9 Inches deep. So then, if you extend the Compaſſes from 12 to 15, that Extent, turn'd twice over from 25 Feet, the Length, will reach to 39 Feet, the Content.

§ III.

§ III. *Of a Triangular* PRISM.

A Priſm is a Solid contained under ſeveral Plains, and having its Baſes like, equal, and parallel. The ſolid Content of a Priſm (whether triangular of multangular) is found by multiplying the Area of the Baſe into the Length or Height, and the Product is the ſolid Content.



Let ABCDEF be a triangular Priſm, each Side of the Baſe being 15.6 Inches, the Perpendicular thereof C *a* is 13.51 Inches, and the Length of the Solid 19.5 Feet.

Multiply the Perpendicular of the Triangle 13.51 by half the Side 7.8, and the Product is 105.378, the Area of the Baſe; which multiply by the Length 19.5, and the Product is 2054.871, which divide by 144, and the Quotient is 14.27 Feet *fere*, the ſolid Content.

13 51
7 8
10808
9457
105.378
19.5
526890
948402
105378
2054.8710

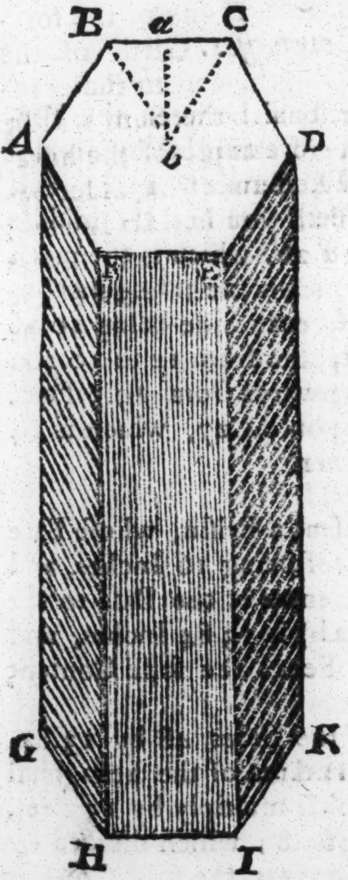
144)2054.87(14.27
144
614
576
388
288
1007
1008

By Scale and Compaſſes.

Firſt, find a mean Proportional between the Perpendicular and Half-ſide, (as before taught) by divideing the Space upon the Line, between 13.51 and 7.8 into two equal Parts; ſo ſhall you find the middle Point between them to be at 10.26, which is the mean Proportional ſought: By this means the triangular Solid is brought to a ſquare one, each Side being 10.26 Inches. Then extend the Compaſſes from 12 to 10.26; that Extent, turn'd twice downwards from 19.5 Feet, the Length will at laſt fall upon 14.27, which is 14 Feet and a little above a Quarter.

Let ABCDEFGHIK repreſent a Priſm, whoſe Baſe is a Hexagon, each Side thereof being 16 Inches, and the Perpendicular from the Centre of the Baſe to the Middle of one of the Sides, (ab) is 13.84 Inches, and the Length of the Priſm is 15 Feet; the ſolid Content is required.

Multiply half the Sum of the Sides 48 by 13.84, and the Product is 664.32, the Area of the hexagonal Baſe, (by § VIII. *p.* 86.) which multiply by 15 Feet, the Length, the Product is 9964.8; which divided by 144, the Quotient will be 69.2 Feet, the ſolid Content required.



13.84
48

11072
5536

664.32 Area of Baſe.
15

332160
66432

144)9964.80(69.2
864

1324
1296

288
288

...

By Scale and Compaſſes.

Firſt, find a mean Proportional between the Perpendicular, and half the Sum of the Sides; that is, divide the Space between 13.84 and 48, and the middle Point will be 25.77. Then extend the Compaſſes from 12 to 25.77; that Extent will reach (being twice turn'd over) from 15 Feet, the Length, to 69.2 Feet, the Content.

To find the ſuperficial Content of any of the forementioned Solids, you muſt take the Girth of the Piece, and multiply by the Length, and to that Product add the two Areas of the Baſes, the Sum will be the whole ſuperficial Content. Example of the hexagonal Priſm laſt-mentioned: The Sum of the Sides being 96, and the Length 15 Feet, that is, 180 Inches; which multiply'd by 96, the Product is 17280 ſquare Inches; to which add twice 664.32, the Areas of the two Baſes, and the Sum is 18608.64, the Area of the Whole, which is 129.22 Feet.

```
         180
          96
        ----
        1080
       1620
       -----
       17280
         664.32
         664.32
       ----------
  144)18608.64(129.22
       ----
      420
      1328
        326
         384
         ---
          96
```

The ſuperficial Content of the whole Solid is 129.22 Feet.

By Scale and Compaſſes.

Extend the Compaſſes from 144 to 180; that Extent will reach from 96 to 120 Feet. Then, to find the Area of the Baſe, extend the Compaſſes from 144 to 13.84; that Extent will reach from 48 to 4.6 Feet; add 120 Feet, and twice 4.6 Feet, and it makes 129.2 Feet, the ſuperficial Content, as before.

The Demonſtration of thoſe laſt Solids will be the ſame as in the firſt Section; for as in that, ſo in theſe, the Area of the Baſe is multiply'd into the Length to find the Content, and the ſame Reaſon is given for one as for the other.

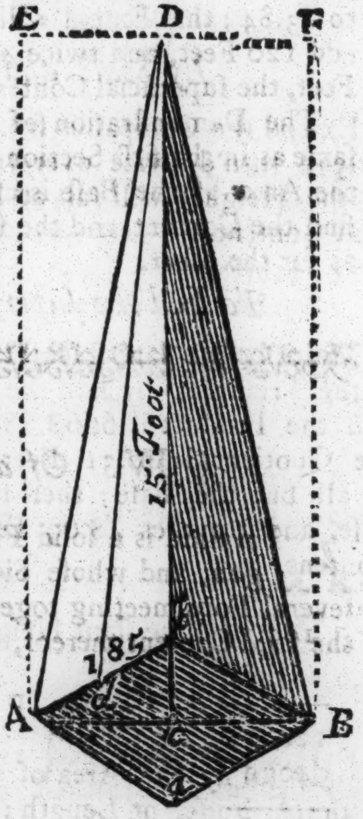
* * *

§ IV. *Of a* PYRAMID.

A Pyramid is a ſolid Figure, whoſe Baſe is a Polygon, and whoſe Sides are plain Triangles, their ſeveral Tops meeting together in one Point. To find the ſolid Content thereof, this is

The RULE.

Multiply the Area of the Baſe by a third Part of the Altitude, or Length; and the Product is the ſolid Content of the Pyramid.



Let ABD be a ſquare Pyramid, each Side of the Baſe being 18.5 Inches, and the perpendicular Height CD is 15 Feet: Multiply 18.5 by 18.5, and the Product is 342.25, the Area of the Baſe in Inches; which multiply'd by 5, a third Part of the Height, and the Product is 1711.25; this divided by 144, the Quotient is 11.88 Feet, the ſolid Content.

```
        18.5
        18.5
       ------
         925
       1480
       185
       ------
      342.25  Area of the Baſe.
           5
       ------
144)1711.25(11.88 Content.
```

F.	I.	Pts.	
1	6	6	
1	6	6	
1	6	6	
	9	3	
		9	3
2	4	6	3
			5
11	10	7	3

By Scale and Compaſſes.

Extend the Compaſſes from 12 to 18.5 Inches, that Extent, turn'd twice over from 5 Feet, (a third Part of the Height) will fall at laſt upon 11.88 Feet, the ſolid Content.

To find the ſuperficial Content.

Multiply the ſlant Height (or Perpendicular of one of the Triangles) by half the Periphery of the Baſe 37, and the Product is 6668.88, which divided by 144, the Quotient is 46.31 Feet, the ſuperficial Content of all but the Baſe; then to that add 2.38 Feet the Baſe, and it makes 48.69 Feet, the whole ſuperficial Content.

180.24 The ſlant Height d D.
37
126168
54072
144)6668.88(46.31
576
908
864
448
432
168
144
24

2.38
48.69 The whole Content.

144)342.25(2.38
288
542
432
1105
1152

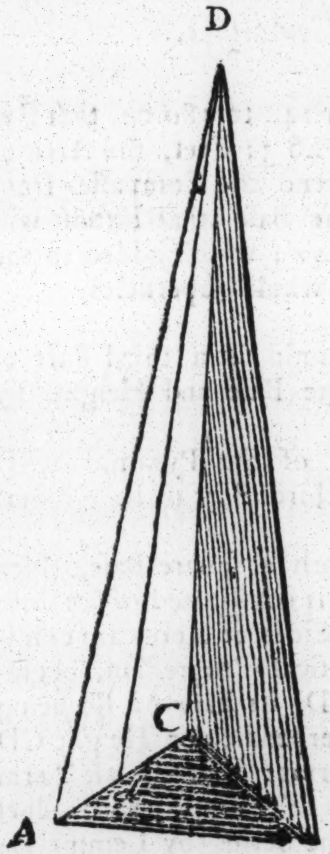
By Scale and Compaſſes.

Extend the Compaſſes from 144 to 180.24, that Extent will reach from 37 to 46.31 Feet, the Area of the four Triangles; and extend the Compaſſes from 144 to 18.5 (one Side of the Baſe) that Extent will reach from 18.5 to 2.38 *fere*; which added to the other, the Sum is 48 69, the whole Superficies.

Demonſtration. Every Pyramid is a third Part of the Priſm, that hath the ſame Baſe and Height (by *Eucl.* 12.7.)

That is, the ſolid Content of the Pyramid ABD (in the laſt Figure) is one third Part of its circumſcribing Priſm ABEF.

For every Pyramid that hath a ſquare Baſe, (ſuch as Aa Bb in the laſt Figure) is conſtituted of an infinite Series of Squares, whoſe Sides or Roots are continually increaſing in arithmetical Progreſſion, beginning at the Vertex or Point D, its Baſe Aa Bb being the greateſt Term, and its perpendicular Height CD is the Number of all the Terms: But the laſt Term multiply'd into the Number of Terms, the Product will be triple the Sum of all the Series (by Lemma 3); conſequently $\frac{NLL}{3} = S$. And S is equal to the ſolid Content of the Pyramid. From hence it will be eaſy to conceive, that every Pyramid is $\frac{1}{3}$ of its circumſcribing Priſm, (that is, of a Priſm of equal Baſe and Altitude) what Form ſoever its Baſe is of, *viz.* whether it be ſquare, triangular, pentangular, *&c.* You may very eaſily prove a triangular Pyramid to be a third Part of a Priſm of equal Baſe and Altitude, by cutting a triangular Priſm of Cork, and then cut that Priſm into three Pyramids, by cutting diagonally, as I have ſeveral times done, to ſatisfy myſelf and others.



Let ABCD be a triangular Pyramid, each Side of the Baſe being 21.5 Inches, and its perpendicular Height 16 Feet; the Content, ſolid and ſuperficial, is required.

Firſt, find the Area of the Baſe, by multiplying half the Side by the Perpendicular let fall from the Angle of the Baſe to the oppoſite Side; which Perpendicular will be found to be 18.62; the Half thereof is 9.31, multiply'd by 21.5, the Product is 200.165 Inches, the Area of the Baſe. Then, becauſe the Altitude 16 cannot exactly be divided by 3, therefore I take the third Part of 200.165, which is 66.72, and multiply it by 16, and the Product is 1067.52, which divided by 144, the Quotient is 7.41 Feet, the ſolid Content.

			F.	I.	Pts.	
9.31	Half the Perpend.					
21.5	The Side.	Side	1	9	6	
		Half. Perp.		9	4	
4655						
931			1	4	1	6
1862					7	2
3)200.165	Area Baſe.	Area Baſe	1	4	8	8
						4
66.72	A third Part.					
16	Height.		5	6	10	8
						4
40032						
6672			3)22	3	6	8
144)1067.52(7.41	Solid Cont.	Content	7	5	2	2
1008						
595						
576						
192						
144						
48						

In caſting this up by Feet and Inches, inſtead of multiplying by 16 the Height, I break 16 into two ſuch Numbers, as, being multiplied together, the Product may be 16, *viz.* into 4 and 4, and multiply firſt by one, and then the other; a third Part of the laſt Product is the Content.

By Scale and Compaſſes.

Firſt, find a geometrical mean Proportional, (as before directed) by dividing the Space between 21.5 and 9.31 into two equal Parts, and you will find the middle Point at 14.15, which is the mean Proportional ſought. Then extend the Compaſſes from 12 to

14.15

14.15, that Extent (turn'd twice over from 16 Feet, will fall at laſt upon 22.23; a third Part thereof is 7.41 Feet, the Content.

To find the ſuperficial Content.

Multiply the ſlant Height (or Perpendicular of one of the Triangles) by half the Periphery of the Baſe, and to that Product add the Area of the Baſe, the Sum is the whole ſuperficial Content.

```
               192.1  Inches, the ſlant Height d D.
Half Periph.   32.25=21.5+10.75.
               -----
                9605
               3842
              3842
             5763
             --------
             6195.225 Inches, the Area of all but the Baſe.
              200.165 Area of the Baſe add.
             --------
        144)6395.390(44.41 Feet, the whole Content.
            576
            ---
             635
             576
             ---
              593
              576
              ---
               179
               144
               ---
                35
```

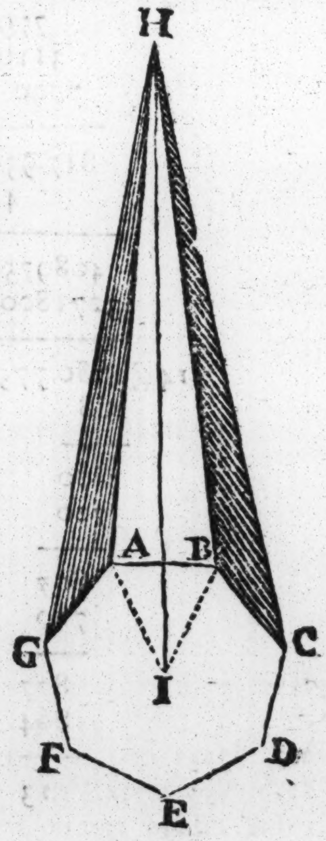
By Scale and Compaſſes.

Extend the Compaſſes from 144 to 192.1, that Extent will reach from 32.25 (half the Periphery of the Baſe) to 43.02 Feet, the Content of the upper Part.

And extend the Compaſſes from 144 to half the Perpendicular 9.31, that Extent will reach from the Side 21.5 to 1.39 Feet, the Area of the Baſe; which, added to the other, makes 44.41 Feet, the Content of the Whole.

Let A B C D E F G H be a Pyramid, whoſe Baſe is a Heptagon, each Side thereof being 15 Inches, and the Perpendicular of the Heptagon is 15.58 Inches, and the perpendicular Height of the Pyramid HI is 13.5 Feet; the Content ſolid and ſuperficial is required.

Multiply 15.58 (the Perpendicular) by 52.5, (half the Sum of the Sides of the Heptagon) and the Product is 817.95, which multiply'd by 4.5, *viz.* ⅓ of the Height, and the Pro- is 3680.775.



Then divide this laſt Product by 144, and the Quotient is 25.56 Feet, the Content.

```
     15.58 The Heptagon's Perpend.
      52.5 The Half Sum of the Sides.
     -----
      7790
     3116
    7790
   -------
   817.950
       4.5 A third Part of the Height.
  --------
  4089750
 3271800
 --------
144)3680.7750(25.56 Solid Feet.
    288
    ---
    800
    720
    ---
     807
     720
     ---
      877
      864
      ---
       13
```

By Scale and Compaſſes.

Firſt, find a geometrical mean Proportional between 15.58 and 52.4, (as is before directed) which you will find to be 28.06; then extend the Compaſſes from 12 to 28.06, that Extent will reach from 4.5 (twice turn'd over) to 25.56 Feet.

To find the ſuperficial Content.

Multiply the Height taken from the Middle of one of the Sides of the Baſe 162.75 Inches, by the Half-Sum of the Sides 32.5 Inches, and the Product is 8544.375; which divided by 144, the Quotient is 59.335 Feet, the Content of the upper Part.

162.75
52.5
81375
32550
81375

144)817.95(5.68
979
1155
3

144)8544.375(59.335 Feet.
1344
483
517
855
135

5.68 Baſe add.
65.015 The whole Content.

By Scale and Compaſſes.

Extend the Compaſſes from 144 to 162.75, that Extent will reach from 52.5 to 59.335 Feet.

And extend the Compaſſes from 144 to 15.58, the Perpendicular of the Heptagon, that Extent will reach from 5.25 to 5.68 Feet, the Content of the Baſe; which add to the former, the Sum is 65.015, the whole ſuperficial Content.

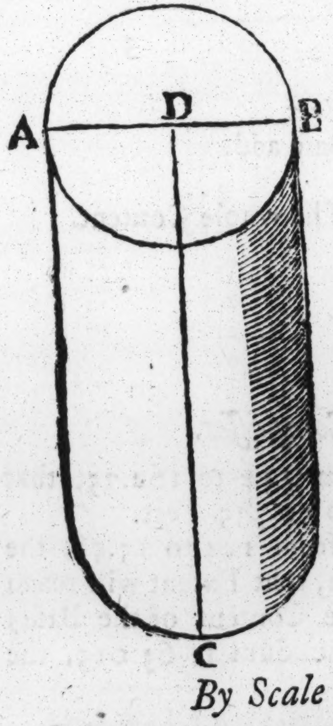
§ V.

§ V. *Of a* CYLINDER.

A Cylinder is a round Solid, having its Baſes circular, equal and parallel, in form of a Rolling-ſtone uſed in Gardens. To find the ſolid Content thereof, this is

The RULE.

Multiply the Area of the Baſe by the Length, and the Product is the ſolid Content.



Let A B C be a Cylinder, whoſe Diameter AB is 21.5 Inches, and the Length CD is 16 Feet; the ſolid Content is required.

Firſt, ſquare the Diameter 21.5, and it makes 462.25; which multiply by .7854, and the Product is 363.05115. Then multiply this by 16, and the Product is 5808.8164. Divide this laſt Product by 144, and the Quotient is 40.34 Feet, the ſolid Content.

By Scale and Compaſſes.

Extend the Compaſſes from 13.54 to 21.5, the Diameter, that Extent (turn'd twice over from 16, the Length) will at laſt fall upon 40.34, the ſolid Content.

To

To find the ſuperficial Content.

Firſt, (by Chap. I. Sect. IX. Prob. 2.) find the Circumference of the Baſe 67.54, which multiply by 16, the Product is 1080.64; which divided by 12, the Quotient is 90.05 Feet, the curve Surface; to which add 5.04 Feet, the Sum of the two Baſes, and the Sum is 95.09 Feet, the whole ſuperficial Content.

```
     67.54                              363.05
        16                                   2
    ------                            --------
     40524                       144)726.10(5.04
     6754                             ------
    -------    90.05 } Add.              610
12)1080.64      5.04 }                   ---
    -------    -----                      34
     90.05     95.09
```

By Scale and Compaſſes.

Extend the Compaſſes from 12 to 67.54, (the Circumference) that Extent will reach from 16 (the Length) to 91.05 Feet, the curve Surface.

And extend the Compaſſes from 12 to 21.5, (the Diameter) that Extent (turn'd twice from .7854) will at laſt fall upon 2.52 Feet, the Area of one Baſe; which doubled is 5.04; this, added to the curve Surface, makes 95.09 Feet, the whole ſuperficial Content.

Demonſtration. The ſolid Content of every Cylinder is found, by multiplying the Area of its Baſe into its Height, as aforeſaid: For every right Cylinder is only a round Priſm, being conſtituted of an infinite Series of equal Circles; that of its Baſe, or End, being one of the Terms, and its Height CD (in the former Figure) is the Number of all the Terms. Therefore the Area of its Baſe AB being multiply'd into

CD, will be its Solidity (by Lemma I.). Let D=AB, and H=CD

Then .7854 DD×H=its Solidity.

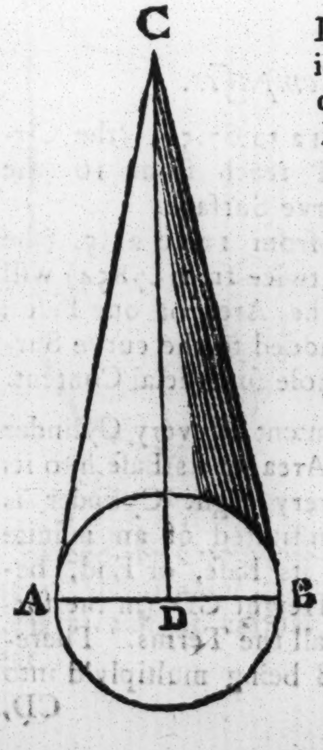
§ VI. *Of a* CONE.

A Cone is a Solid, having a circular Baſe, and growing ſmaller and ſmaller, till it ends in a Point which is called the Vertex, and may be nearly repreſented by a Sugar-loaf. To find the Solidity thereof, this is

The RULE.

Multiply the Area of the Baſe by a third Part of the perpendicular Height, and the Product is the ſolid Content.

Let ABC be a Cone, the Diameter of whoſe Baſe AB is 26.5 Inches, and the Height of the Cone DC is 16.5 Feet: Firſt, ſquare the Diameter 26.5, and it is 702.25, which multiply by .7854, and the Product is 551.54715; which multiply by 5.5, and the Product is 3033.47825; which divided by 144, the Quotient is 21.07 *fere*, the ſolid Content of the Cone.



26.5 The Diameter.
26.5

1325
1590
530

702.25 The Square.
.7854

280900
351125
561800
491575

551.54|715 Area of the Baſe.
5.5 A third Part of the Height.

275770
275770

144)3033.47|0(21.06 Feet the Content.

153
947

83

By Scale and Compaſſes.

Extend the Compaſſes from 13.54 to 26 5, the Diameter, that Extent turned twice over from 5.5, (a third Part of the Height) it will at laſt fall upon 210.6 Feet, the Content.

To find the ſuperficial Content.

Multiply half the Circumference 41.626 by the ſlant Height A C 198.46, and the Product is 8261.09596; which, divided by 144, the Quotient is 57.37 *fere*, the curve Surface; to which add the Baſe, the Sum is 61.2, the ſuperficial Content.

41.626 The half Circumference of the Baſe.
198.46 The ſlant Height.

249756
166504
333008
374634
41626

144)8261.09596(57.37 Feet *fere.*
3.83 The Baſe add.
1061
530
989
61.20 The whole Content.

144)551.54(3.83
1195
434
2

By Scale and Compaſſes.

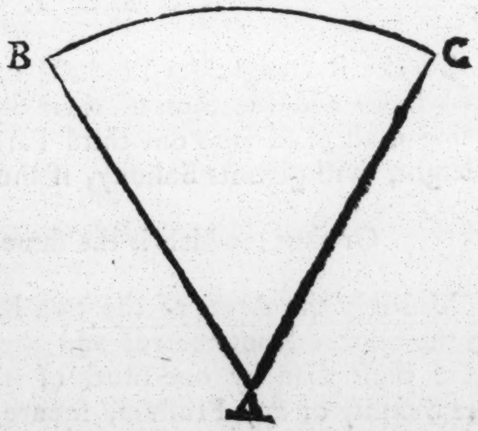
Extend the Compaſſes from 144 to 198.46, that Extent will reach from 41.626 to 57.37 Feet, the curve Surface.

And extend the Compaſſes from 12 to 26.5 the Diameter; that Extent, turned twice over from .7854, will at laſt fall upon 3.83 Feet, the Baſe; which added to 57.37, the Sum is 61.2 Feet, the ſuperficial Content.

Demonſtration. Every Cone is the third Part of a Cylinder of equal Baſe and Altitude. The Truth of this may eaſily be conceived, by only conſidering, that a Cone is but a round Pyramid; and therefore it muſt needs have the ſame Ratio to its circumſcribing Cylinder, as the ſquare Pyramid hath to its circumſcribing Parallelopipedon, *viz.* as 1 to 3. However, to make it yet clearer, let it be farther conſidered, That

Every right Cone is conſtituted of an infinite Series of Circles, whoſe Diameters do continually increaſe in Arithmetical Progreſſion, beginning at the Vertex, or Point C, the Area of its Baſe AB being the greateſt Term, and its perpendicular Height DC, the Number of all the Terms; therefore the Area of the Circle of the Baſe, multiplied by a third Part of the Altitude DC, will be the Sum of all the Series, equal to the Solidity of the Cone, by Lemma III.

The curve Superficies of every right Cone is equal to half the Rectangle of the Circumference of its Baſe into the Length of its Side.



For the curve Suface of every right Cone is equal to the Sector of a Circle whoſe Arch BC is equal to the Periphery of the Baſe of the Cone, and Radius A B equal to the ſlant Side of the Cone. Which will appear very evident, if you cut a Piece of Paper in the Form of a Sector of a Circle, as A B C, and bend the Sides A B and AC together, till they meet, and you will find it to form a right Cone.

I have omitted the Demonſtrations touching the Superficies of all the foregoing Solids, becauſe I thought it needleſs, they being all compoſed of Squares, Parallelograms, Triangles, &c. which Figures are all demonſtrated before. And if the Area of all ſuch Figures as compoſe the Solid, be found ſeverally, and added together, the Sum will be the ſuperficial Content of the Solid.

§ VII. *Of the Fruſtum of a* PYRAMID.

A Fruſtum of a Pyramid is the remaining Part, when the Top is cut off by a Plain Parrallel to the Baſe. To find the Solid Content thereof, there are ſeveral Rules.

RULE I.

To the Rectangle (or Product) of the Sides of the two Baſes add the Sum of their Squares; that Sum, being multiplied into one-third Part of the Fruſtum's Height, will give its Solidity, if the Baſes be ſquare.

Or thus; which is the ſame in Effect:

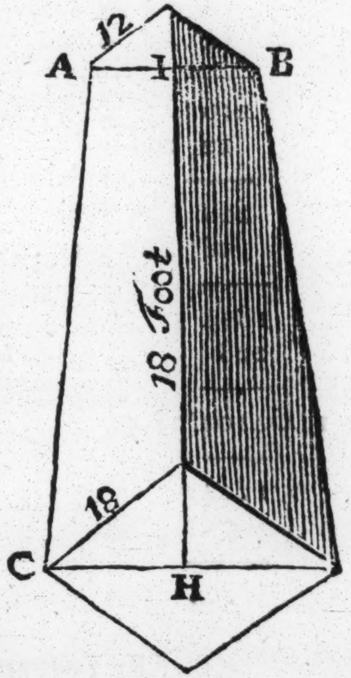
Multiply the Areas of the two Baſes together, and to the Square Root thereof add the two Areas; that Sum multiplied by one-third of the Height, gives the Solidity of any Fruſtum, ſquare or multangled.

RULE II.

To the Rectangles of the Sides of the two Baſes, add one-third Part of the Square of their Difference; that Sum being multiplied into the Height, will produce the Solidity, if the Baſes be Squares: But if they be triangular or multangular, the ſaid Rectangle of the Sides, with the third Part of the Square of their Difference, will be the Square of a mean Side; and the Square Root thereof will be ſuch a mean Side as will reduce the tapering Solid to a Priſm equal thereunto.

Example. Let ABCD be the Fruſtum of a ſquare Pyramid, the Side of the greater Baſe 18 Inches, and the Side of the leſſer 12 Inches, and the Height 18 Feet; the Solidity thereof is required.

First, multiply the two Sides together, 18 by 12, and the Product is 216, and the Difference of the Sides is 6, whoſe Square is 36, a third Part thereof is 12, which added to 216, the Sum is 228 Inches, the Area of a mean Baſe; which multiplied by 18 Feet the Length, the Product is 4104; this divided by 144, the Quotient is 28.5 Feet the Content.



Or, by the firſt Rule, thus: The Square of 18 is 324, and the Square of 12 is 144, and the Rectangle of 18 by 12 is 216; the Sum of theſe three is 684, which multiplied by 6, the Product is 4104; which divided by 144, the Quotient is 28.5 Feet, the ſame as before.

See the Work both Ways.

18 6 Diff.
12 6
216 3)36 Square
12 add—
12 A Third.
228 The Sum.
18 The Height
1824
228
144)4104(28.5
1224
720
...

18 12
18 12
324 Sq. 144 Sq.
144
216
684 The Sum.
6 A 3d of the Height.
144)4104(28.5 Feet.
1224
720
...

By Feet and Inches thus:

	F.	I.	I.
Mult.	1	6	6
by	1		6
Prod.	1	6	3)36 q.
add	0	1	12
Mult.	1	7	
by	18	0	Height.
	18	0	
	9	0	
	1	6	
Cont.	28	6	

Or thus:

F.	I.	
2	3	Sq. of the greater.
1	6	The Rectangle.
1	0	Square of the leſs.
4	9	Trip of a mean Ar.
6	0	a 3d of the Height.
286		

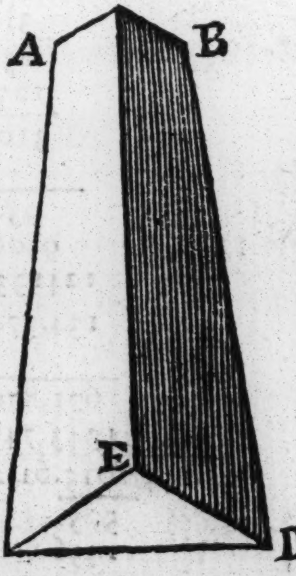
To

To find the ſuperficial Content.

The Perimeter of the greater Baſe is 72, and the Perimeter of the leſſer Baſe is 48; add both the Perimeters together, the Sum is 120; the Half thereof is 60; which multiplied by 18 Feet, the Product is 1080; this divided by 12, the Quotient is 90 Feet; to which add the two Baſes 2.25 Feet, and 1 Foot, the Sum is 93.25 Feet, the whole ſuperficial Content.

18	12	18 The Height.
4	4	60
72	48	12)1080
48		90 Feet.
2)120		2.25 The greater Baſe.
60		1 The leſſer Baſe.
		93.25 The Sum.

Again; Let A B C be the Fruſtum of a triangular Pyramid, each Side of the greater Baſe 25 Inches, and each Side of the leſſer Baſe 9 Inches, and the Length 15 Feet; the ſolid Content thereof is required.



By the ſecond Rule, multiply 25 by 9, and the Product is 225; and the Difference between 25 and 9 is 16; which, ſquared, makes 256, a third Part thereof is 85.333, which added to 225, the Sum is 310.333; and this multiplied by

433, the Product is 134.374, &c. which is the Area of a mean Baſe; and that multiplied by 15 Feet, the Length, the Product is 2015.610; which divided by 144, the Quotient is 13 99 Feet, the Solidity.

Or thus, by the latter Part of the firſt Rule: Find the Area of the greater Baſe, which you will find to be 270.625, and the Area of the leſſer Baſe will be 35.073; theſe two Areas multiplied together, the Product is 9491.630625; the Square Root thereof is 97.425; to which add the two Areas, and the Sum is 403.123; which multiplied by a third Part of the Length 5, the Product is 2015.615; and that divided by 144, the Quotient is 13.99 Feet, as before.

See the Working of both.

25 25
9 9
Product 225 16 Diff.
16
96
16
3)256 The Square.
85.333 A third Part.
225 Add.
310.333
.433 Tabular Numb. (vid. p. 89)
930999
930999
1241332
134.374189 Mean Area.
15 Length.
671.870945
1343.74189
144)2015.61|2835(13.99 Feet.
575
1436
1401
105

25
25

125
50

625 Square.
433

1875
1875
2500

270.625 Area.

9
9

81 Square.

433
81

433
3464

35073

270.625
35.073

811875
1894375
13531250
811875

9491.630625

. .
9491.630625(97.425
81

187)1391
1309

1944)8263
7776

19482)48706
38964

194845)974225
974225

....

270.625 Greater Area.
97.425 The mean Proportional.
35.073 The leſſer Area.

403.123 The Triple of a mean Area.
5 A third Part of the Height.

144)2015.615)13.99 Feet, the Solidity.

575
1436
1401

105

In finding the Area of the triangular Baſe, I multiply by .433, becauſe that is the Area of the equilateral Triangle, when the Side thereof is 1. A Table of the Areas, or Multipliers, for finding the Areas of Polygons, you'll find in *p.* 89.

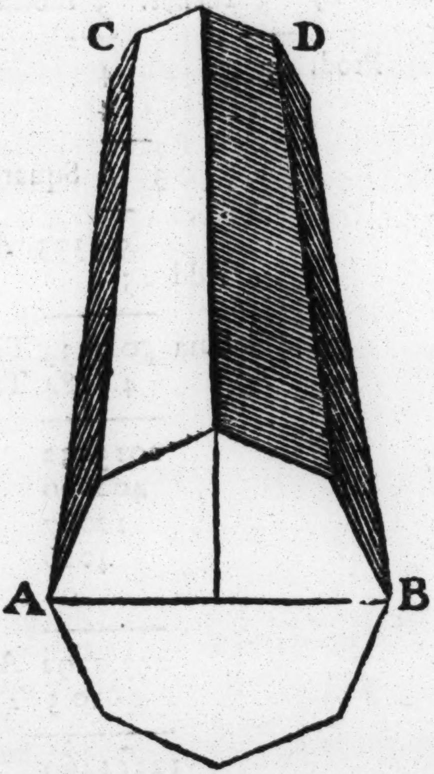
Multiply the Square of the Side by the tabular Number, and the Product is the Area of the Polygon.

To find the ſuperficial Content.

The Perimeter of the greater Baſe is 75, and the Perimeter of the leſſer Baſe is 27; the Sum of both is 102, and the half Sum is 51; which multiplied by 15 Feet, the Product is 765; which divided by 12, the Quotient is 63.75; to which add the Sum of the two Baſes 2.12 Feet, and the Sum is 65.87 Feet, the whole ſuperficial Content.

Note, That 51 ſhould have been multiplied by the ſlant Height, but the Difference it would make is but .06 of a Foot, which is inconſiderable.

Again: Suppoſe ABCD to be the Fruſtum of a Pyramid, having an octagonal Baſe, each Side therereof being 9 Inches, and each Side of the leſſer Baſe 5 Inches, and the Length or Height 10.5 Feet, the Solidity is required.



By the ſecond Rule multiply the greater Side 9 by the leſſer Side 5, and the Product is 45; then the Difference between 9 and 5 is 4, which ſquared makes 16; a third Part thereof is 5.3333, which added to 45, the Sum is 50.3333; multiply this laſt by the Number in the Table 4.8284, and the Product is 243.0292, the Area of a mean Baſe; which multiplied by the Height 10.5 Feet, the Product is 2551.8066; then divide this laſt Product by 144, and the Quotient is 17.72 Feet, the ſolid Content.

See the Work.

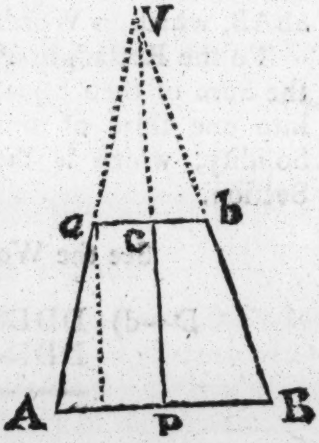
Mult-

```
Mult.  9 Inches.   9 from the greater Side.
  by   5 Inches.   5 ſubtract the leſſer.
      ——          ——
Prod. 45           4
                   4
                  ——
                3 16 Squared.
                  —
                  5.3333 A third Part.
             Add 45
                 ———————
             Sum 50.3333 The Sq. of a mean Side.
                  4.8284 Tabular Number, p. 89.
                 ———————
                 2013332
                  402666
                   10067
                    4026
                     201
                 ———————
                243.0292 A mean Area.
                    10.5 The Height.
                ————————
                12151460
               2430292
               —————————
       144)2551.80|660(17.72
           144
           ———
           1111
           1008
           ————
            1038
            1008
            ————
             300
             288
             ———
              12
```

To find the ſuperficial Content.

The Perimeter of the greater Baſe is 72, and the Perimeter of the leſſer Baſe is 40, and their Sum is 112; the Half thereof is 56, which multiply'd by the Height 10.5 Feet, and the Product is 588; which divided by 12, the Quotient is 49 Feet; to which add the Sum of the two Baſes, 3.55, and the Sum is 52.55 Feet, the whole ſuperficial Content.

Demonſtration. From the Rules delivered in the IVth and VIth Sections, the two foregoing Rules may eaſily be demonſtrated.



Suppoſe a ſquare Pyramid, ABV, to be cut by a Plane at *a* b, parallel to its Baſe AB, and it were requir'd to find the Solidity of the Fruſtum, or Part *a* b AB. Let there be given

D=BA, the Side of the greater Baſe.
d=b*a*, the Side of the leſſer Baſe.
H=CP, the perpendicular Height.

Firſt, 1 $D-d : H :: d : \frac{dH}{D-d}$ =VC by the Figure.
Then 2 $DD \times \frac{H+VC}{3}$ =the whole Pyramid BVA. by Section the IVth.
And 3 $dd \times \frac{1}{3}VC$ =the Pyramid *a* Vb cut off.

Then in the 2d and 3d Steps, if, inſtead of VC, you take $\frac{dH}{D-d}$ equal to it by the firſt Step, it will be,

viz. 1. 2. | 4 | $\frac{DDDH}{3D-3d}$ = the whole Pyramid BVA.

and 1. 3. | 5 | $\frac{dddH}{3D-3d}$ = the Pyramid *a* V b.

4.—5. | 6 | $\frac{DDDH-dddH}{3D-3d}$ = the Fruſtum *a*bAB.

And by dividing DDD—ddd by D—d, and then multiplying the Quotient by $\frac{1}{3}$ H, the laſt Step will be reduced to $DD+Dd+dd : \times \frac{1}{3} H$ = the Fruſtum *a*bAB, which in Words is thus;

To the Rectangle of the Sides of the two Baſes add the Sum of their Squares; that Sum being multiply'd into one third of the Fruſtum's Height, will give its Solidity, which is the ſame as the firſt Rule of this Section.

See the Work of the Diviſion.

$$
\begin{array}{rl}
D-d) & DDD-ddd\ (DD+Dd+dd \\
 & DDD-DDd \\
\hline
 & DDd-ddd \\
 & DDd-Ddd \\
\hline
 & Ddd-ddd \\
 & Ddd-ddd \\
\hline
 & 0\quad 0
\end{array}
$$

The ſame Reaſon will hold good for all Fruſtums of Pyramids or Cones, whether the Baſe be triangular or multangular, becauſe the Squares of the Sides of any Figure, or the Squares of the Diameters of Circles, are proportional to the Area, which proves the latter Part of the ſaid firſt Rule.

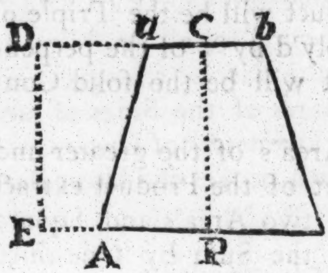
Again, to prove the ſecond Rule.

Suppoſe	1	$x = D - d$. And $F =$ the Fruſtum.
then	2	$DD + Dd + dd = \frac{3F}{H}$ by the laſt.
1 ⊛ 2	3	$xx = DD - 2Dd + dd$.
2 — 3	4	$3Dd = \frac{3F}{H} - xx$.
4 — 3	5	$Dd = \frac{F}{H} = \frac{1}{3}xx$. Or $Dd + \frac{1}{5}xx = \frac{F}{H}$
5 × H	6	$.\overline{Dd + \frac{1}{3}xx}: \times H = F$, the Fruſtum *a*bAb.

Which in Words is thus:

To the Rectangle of the Sides of the two Baſes add one third Part of the Square of the Difference of the ſaid Sides, and multiply the Sum by the Height of the Fruſtum, the Product is the Solidity of the Fruſtum.

The ſuperficial Contents of Fruſtums (all but the Baſes) are compos'd of Trapeziums, ſo many as the Fruſtum has Sides. As the ſquare Fruſtum *a*bAb, in the laſt Figure, is compos'd of four Trapeziums, having the two upper, and alſo the two longer Angles equal; if therefore the Trapezium *a*bAB be cut in two by the Line CP, and the two Pieces laid together, the Line bB upon the Line *a*A, the narrow End of the one to the broad End of the other, it will form a right-angled Parallelogram, as is plain by the Figure an-



nex'd; the Parallelogram DCEP being equal to the Trapezium *ab*AB; becauſe the Side D*a* is equal to PB, and EA is equal to *a* C. Therefore, to find the Area of the Trapezium, add half the Side *ab* to half the Side AB, and it makes DC or EP; which multiply by the Height PC, the Product is the Area of the Parallelogram DCEP, equal to the Trapezium *ab*AB; then if that be multiply'd by the Number of Trapeziums, the Product will be the ſuperficial Content of the Fruſtum, wanting the Baſes. Or, if the whole Perimeter of the greater Baſe be added to the Perimeter of the leſſer Baſe, and half the Sum multiply'd by the Height, the Product will be the ſuperficial Content of all the Trapeziums at once.

Note, That half the Sum of the Perimeters ſhould be multiply'd by the ſlant Height, up the Middle of one of the Trapeziums; but in the foregoing Examples I have multiply'd by the perpendicular Height, becauſe the Difference is very inconſiderable.

§ VIII. *Of the Fruſtum of a* CONE.

A Fruſtum of a Cone, is that Part which remains, when the top End is cut off by a Plain parallel of the Baſe. To find the ſolid Content, the Rules are the ſame in Effect as for the Fruſtum of a Pyramid.

RULE I.

To the Rectangle of the Diameters of the two Baſes add the Squares of the ſaid Diameters, and multiply the Sum by .7854, the Product will be the Triple of a mean Area; which multiply'd by $\frac{1}{3}$ of the perpendicular Height, that Product will be the ſolid Content.

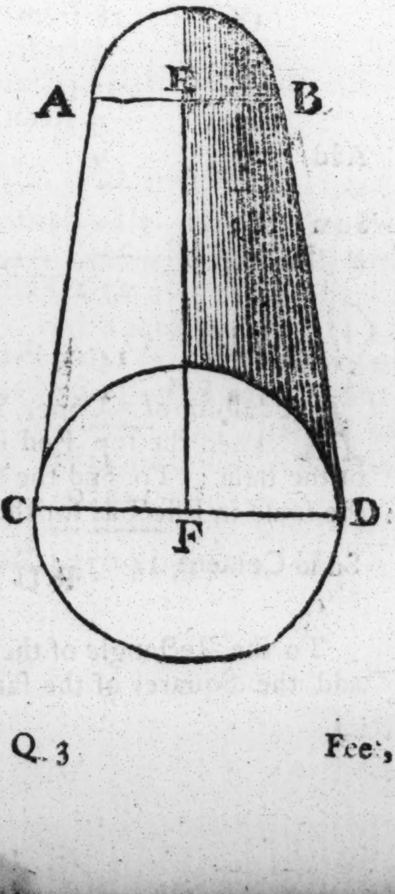
Or thus: Multiply the Area's of the greater and leſſer Baſes together, and out of the Product extract the Square Root, and add the two Area's and Square Root together, and multiply the Sum by One-third of the perpendicular Height, the Product is the ſolid Content.

RULE II.

To the Rectangle of the greater and leſſer Diameters, add one third Part of the Square of their Difference, and multiply the Sum by .7854, the Product is a mean Area; which multiply'd by the perpendicular Height, the Product is the Solidity.

Example. Let ABCD be the Fruſtum of a Cone, whoſe greater Diameter CD is 18 Inches, and the leſſer Diameter AB 9 Inches, and the the Length 14.25 Feet; the ſolid Content is requir'd.

Multiply 18 by 9, and the Product is 162, and the Difference between 18 and 9 is 9, whoſe Square is 81, a third Part is 27; which add to 162, the Sum is 189; this multiply'd by .7854, the Product is 148.44; which divided by 144, the Quotient is 1.03



Feet, the Area of a mean Baſe; which multiply by 14.25 Feet, the Height, the Product is 14.6775 Feet, the ſolid Content.

Or thus, by the firſt Rule.

The Square of 18 (the greater Diameter) is 324, and the Square of 9 (the leſſer Diameter) is 81, and the Rectangle, or Product of 18 by 9, is 162; the Sum of theſe three is 567, which multiply'd by .7854, the Product is 445.3218; which divided by 144, the Quotient is 3.09 Feet, the triple Area of a mean Baſe; this, multiply'd by 4.75 Feet, (a third Part of the Height) and the Product is 14.6775 Feet, the Solidity, the ſame as before.

See the Work.

	18	18 from	.7854
	9	9 ſubtr.	189
	162	9 Rem.	70686
Add	27	9	62832
			7854
Sum	189	3)81 Square	
			144)148.4406(1.03
		27 a Third	144
Height		14.25 Feet	444
Area Baſe		1.03 Feet	432
		4275	12
		14250	
Solid Content		14.6775 Feet.	

```
324 the Square of 18.
162 the Rectangle.
 81 the Square of 9.
———
567 the triple Square of a mean Diameter.

          .7854
            567
         ——————
          54978
         47124
        39270
      ——————————
144)445.32(|18          (3.09
    ——————                4.75
     1332               ——————
    ——————                1545
       36                2163
                        1236
                        ——————
          The Solidity 14.6775
```

To find the ſuperficial Content.

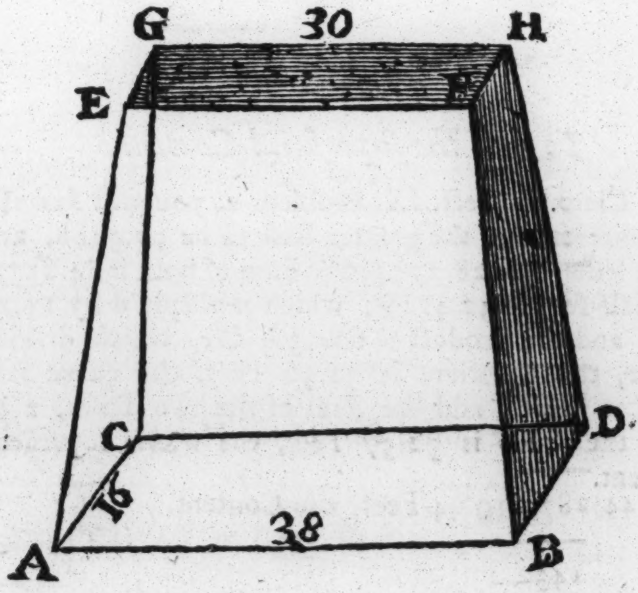
By Chap. I. Sect. IX. Problem 2. you will find the Circumference of the greater Baſe to be 56.5488, and of the leſſer Baſe 28.2744; the Sum of both is 84.8232; the Half-ſum is 42.4116; which multiply'd by 14.25 Feet, and the Product is 604.36, &c. which divided by 12, the Quotient is 50.36 Feet, the curve Surface; to which add the Sum of the two Baſes, 2.21 Feet, the Sum is 52.57 Feet, the whole ſuperficial Content.

§ IX. *To meaſure the Fruſtum of a rectangled* Pyramid, *called a* PRISMOID, *whoſe Baſes are parallel one to another, but diſproportional.*

The RULE.

TO the greateſt Length add half the leſſer Length, and multiply the Sum by the Breadth of the greater Baſe, and reſerve the Product.

Then, to the leſſer Length, add half the greater Length, and multiply the Sum by the Breadth of the leſſer Baſe; and add this Product to the other Product reſerv'd, and multiply that Sum by a third Part of the Height, and the Product is the ſolid Content.



Example. Let ABCDEFGH be a Priſmoid given, the Length of the greater Baſe AB 38 Inches, and its Breadth AC 16 Inches; and the Length of the leſſer Baſe EF is 30 Inches, and its Breadth 12 Inches, and the Height 6 Feet; the ſolid Content is required.

To the greater Length AB 38, add half EF the leſſer Length 15, the Sum is 53; which multiply'd by 16, the greater Breadth, and the Product is 848; which reſerve.

Again, to EF 30, add half AB 19, and the Sum is 49; which multiply by 12, (the leſſer Breadth EG) the Product is 588; to which add 848, (the reſerv'd Product) and the Sum is 1436; which multiply'd by 2, (a third Part of the Height) and the Product is 2872; divide this Product by 144, and the Quotient is 19.94 Feet, the ſolid Content.

38=AB
15=½EF

53
16=AC

318
53

848
588

1436
2=a third Part of the Height.

144)2872(19.94 Feet, the Content.

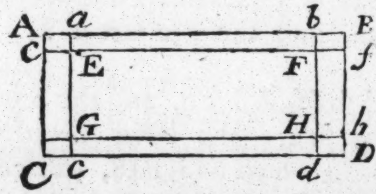
1432
1360
640

64

30=EF
19=½AB
—49
12=EG

588

To prove this Rule, Let us ſuppoſe the Solid cut into Pieces, ſo as to make it capable of being meaſured by the foregoing Rules, thus ; let ABCD repreſent the greater Baſe, and EFGH the leſſer Baſe ; and let the Solid be ſuppos'd to be cut thro' by the Lines, *ac*, *bd*, and *ef*, *gb*, from the Top to the Bottom ; ſo will there be a Parallelopipedon, having its Baſes equal to the leſſer Baſe EFGH, and its Height 6 Feet, equal to the Height of the Solid: Multiply 30 (the Length of the Baſe by 12, the Breadth thereof) and the Product is 360 ; which multiply'd by the Height 6 Feet, and the Product is 2160. Then there are two Wedge-like Pieces, whoſe Baſes are *ab*EF, and GH*cd* ; if theſe two Pieces be laid together, the thick End of one to the thin End of the other, they will compoſe a rectangled Parallelopipedon ; which to meaſure, multiply the Length of the Baſe 30 by its Breadth 2, and the Product is 60 ; which multiply'd by 6, (the Height) the Product is 360. Then there are two other Wedge-like Pieces, whoſe Baſes are *e* E *g* G, and F *f* H *h* ; theſe two laid together, will compoſe a rectangled Parallelopipedon ; to meaſure this, multiply the Length of the Baſe 12 by the Breadth 4, the Product is 48 ; which multiply'd by 6, (the Height) the Product is 288. And laſtly, there are four rectangled Pyramids, at each Corner one ; which to meaſure, multiply the Length of one of the Baſes 4 by its Breadth 2, the Product is 8 ; which multiply'd by 2, (a third Part of the Height) the Product is 16 ; and that multiply'd by 4, (becauſe there are four of them) the Product is 64. Then add all theſe together, and the Sum is 2872 ; and divide by 144, the Quotient is 19.94 Feet, the ſame as before, which ſhews the Rule to be true.



See the Work.

12	30	12	4
30	2	4	2
360	60	48	8
6	6	6	2
2160	360	288	16
360			4
288			
64			64

144)2872(19.94 Feet, the whole Content.

1432
1360
640
64

To find the ſuperficial Content.

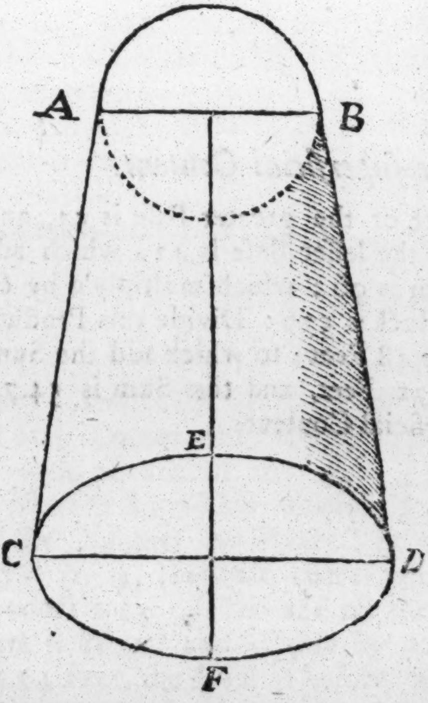
Half the Perimeter or the greater Baſe is 54, and half the Perimeter of the leſſer Baſe is 42; which added together, the Sum is 96; which multiply'd by 6, (the Height) the Product is 576: Divide this Product by 12, the Quotient is 48 Feet; to which add the Sum of the two Baſes 6.72 Feet, and the Sum is 54.72 Feet, the whole ſuperficial Content.

X. *To meaſure a* CYLINDROID; *that is, a Fruſtum of a* Cone, *having its Baſes parallel to each other, but unlike.*

The RULE.

TO the longeſt Diameter of the greater Baſe, add half the longeſt Diameter of the leſſer Baſe, and multiply the Sum by the ſhorteſt Diameter of the greater Baſe, and reſerve the Product.

Then, to the longeſt Diameter of the leſſer Baſe, add half the longeſt Diameter of the greater Baſe, and multiply the Sum by the ſhorteſt Diameter of the leſſer Baſe, and add the Product to the former reſerv'd Sum, and that Sum will be the triple Square of a mean Diameter; which multiply'd by .7854, and that Product multi-ply'd by a third Par of the Height, th Product is the ſoli Content.



Examp. Let ABC be a Cylindroid whoſe Bottom-baſe i an Oval, the tranſ verſe Diameter bein 44 Inches; and th conjugate Diamete 14 Inches; and th upper Baſe is a Circl whoſe Diameter 26 Inches; and th Height of the Fr ſtum is 9 Feet; th Solidity is requir'd.

To 44 (the great Diameter of the low er Baſe) add 13, ha the Diameter of the greater Baſe) the Sum is 57; which multiplied by 14, (the conjugate Diameter of the greater Baſe) the Product is 798; which reſerve. Then to 26 (the Diameter of the leſſer Baſe) add 22, (half the tranſverſe Diameter of the greater Baſe) and the Sum is 48; which multiplied by 26, (the Diameter of the leſſer Baſe) the Product is 1248; to which add the former reſerved Product, the Sum is 2046; which multiplied by .7854, the Product is 1606.9284; which multiplied by 3, (a third Part of the Height) the Product is 4820.7852; which divided by 144, the Quotient is 33.47 Feet the ſolid Content. See the Work.

44=CD
13=half AB
—
57 Sum
14=EF
—
228
57
—
798 Product reſerved.

26=AB
22=Half CD
—
48 Sum.
26=AB.
—
288
96
—
1248
798 add.
—
2046
.7854
—
8184
10230
16368
14322
—
1606 9284
3
—
144)4820 78|52(33.47
500
687
1118
—
110

This Rule being the ſame as that in the laſt Section, the Proof of that may ſerve as ſufficient Proof of this, if what has been before written be well conſidered.

To find the ſuperficial Content.

To the Periphery of the Ellipſis 97 41, add the Periphery of the Circle 81.68, and the Sum is 179.09; the half thereof 89.545, multiplied by 9, the Product is 805.905; which divided by 12, the Quotient is 67.16 Feet, the curve Surface: Then the Area of the Ellipſis is 3.36 Feet, and the Area of the Circle is 3.68 Feet; both which added to the curve Surface, the Sum is 74,2 Feet, the whole ſuperficial Content.

* * *

§ XI. *Of a* SPHERE *or* GLOBE.

A Sphere, or Globe, is a round ſolid Body, every Part of whoſe Surface is equally diſtant from a Point within it, called its Centre; and it may be conceived to be formed by the Revolution of a Semicircle round its Diameter. To find its Solidity, this is

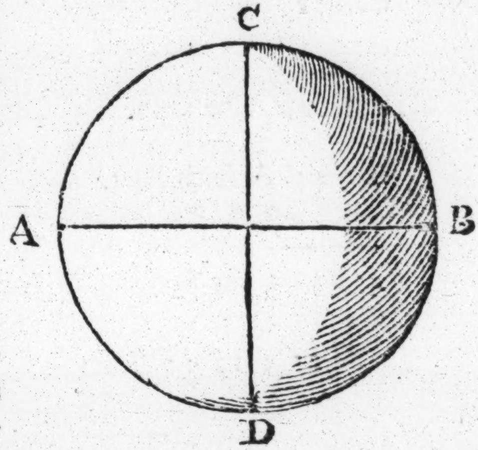
The RULE.

1. Multiply the Axis, or Diameter, into the Circumference, the Product is the ſuperficial Content; which multiply by a ſixth Part of the Axis, the Product is the Solidity.

2. Or thus: As 21 is to 11; ſo is the Cube of the Axis to the ſolid Content.

3. Or, As 1 is to .5236; ſo is the Cube of the Axis to the ſolid Content.

Example.



Example. Let ABCD be a Globe whoſe Axis is 20 Inches, then the Circumf. will be 62.832: Then by the firſt Rule, multiply the Circumference by the Axis, and the Product will be 1256.64, which is the ſuperficial Content in Inches; take a ſixth Part thereof, which is 209.44, (becauſe an exact ſixth Part of 20 cannot be taken) multiply that ſixth Part by 20 (the Axis), and the Product is 4188.8, the Solidity in Inches. Or, if you multiply the ſuperficial Content by the Axis, and take a ſixth Part of the Product, the Anſwer will be the ſame.

Or thus, by the ſecond Rule:

The Cube of the Axis is 8000, which multiplied by 11, the Product is 88000; which divided by 21, the Quotient is 4190.47, the Solidity.

Or, by the third Rule:

If the Cube of the Axis be multiplied by .5236, the Product is 4188 8, the Solidity, the ſame as by the firſt Way. If you divide 4188.8 by 1728, the Quotient is 2.424 Feet.

See the Work.

```
   62.832
       20
 ---------
6)1256.640 The ſuperficial Content.
 ---------
   209.44 A ſixth Part.
       20
   -------
  4188.80 the Solidity in Inches.

      21 : 11 : : 8000
                    11
                  ----
            21)88000(4190.47 The Content.
                 ----
                 40
                 190
                   100
                    160
                    ---
                     13

   1 : .5236 : : 8000
                 8000
            ---------
1728)4188.8000(2.424 Feet, the Solidity.
     -------
      7328
       4160
        7040
        ----
         128
```

Note. If the Axis of a Globe be 1, the Solidity will be .5236; and if the Circumference be 1, the Solidity will be .016887.

By

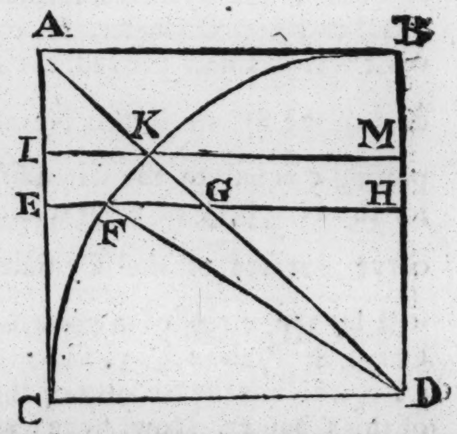
By Scale and Compaſſes.

Extend the Compaſſes from 1 to 20 (the Axis), that Extent (turned three times over from .5236), will at the laſt fall upon 4188.8, the ſolid Content in Inches: Or, extend the Compaſſes from 1728 to 8000, (the Cube of the Axis) that Extent will reach from .5236 to 2.424, the ſolid Content in Feet.

Extend the Compaſſes from 1 to 20 (the Axis), that Extent (turned twice over from 3.1416), will at laſt fall upon 1256.64, the ſuperficial Content in Inches: Or, extend the Compaſſes from 144 to 400, (the Square of the Axis) that Extent will reach from 3.1416 to 8.72, the ſuperficial Content in Feet.

Demonſtration. Every Sphere is equal to a Cone, whoſe perpendicular Axis is the Radius of the Sphere, and its Baſe a Plain equal to all the Surface of it.

For you may conceive the Sphere to conſiſt of an infinite Number of Cones, whoſe Baſes, taken altogether, compoſe the Surface, and whoſe Vertexes meet altogether in the Centre of the Sphere: Hence the Solidity of the Sphere will be gained, by multiplying its Surface by $\frac{1}{3}$ of its Radius.



Let the Square ABCD, the Quadrant CBD, and the right-angled Triangle ABD, be ſuppoſed all three to revolve round the Line BD as an Axis: Then will the Square generate a Cylinder, the Quadrant a Hemiſphere, and the Triangle a Cone, all of the ſame Baſe and Altitude.

Then the Square of EH ($= \square$ FD) $= \square$ FH $+$ $\square$ DH (but DH$=$GH). And ſince Circles are as the Squares of their Diameters, (by *Euclid* 12. 2.) the Circle made by the Revolution of E H muſt be equal to both the Circles made by the Motions of F H and G H.

If you take the Circle made by the Revolution of FH from both, there will remain the Circle made by the Motion of G H, equal to the Ring deſcribed by the Motion of E F. And thus it will always be, where ever you draw the Line EH or IM, *&c.*

Therefore the Aggregate, or Sum, of all the Rings made by the Revolution of the EF's, muſt be equal to that of all the Circles made by the Motion of the GH's, *i. e.* the Diſh-like Solid, formed by the revolving Rings, will be equal to the Cone, formed by the Revolution of the GH's, which are the Elements of the Triangle ABD; that is, the Diſh-like Solid will be as the Cone, $\frac{1}{3}$ of the circumſcribing Cylinder, and conſequently the Hemiſphere muſt be $\frac{2}{3}$ of it: Wherefore the Sphere is $\frac{2}{3}$ of the circumſcribing Cylinder.

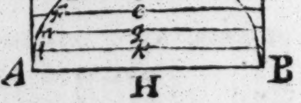
Let the Radius of the Sphere be r $=$ CD, then the Diameter will be 2 r; let the Surface of the Sphere, generated by the revolving Semicircle, be called S, and that of the Cylinder, formed by the Revolution of 2 AC $= 2$ r $=$ Diameter, be called ſ. Wherefore in what was juſt now proved, the Expreſſion for the Solidity of the Sphere in this Notation will be $\frac{1\,rS}{3}$; and putting c equal to the Circumference of the Baſe, or for the Periphery of a great Circle of the Sphere, the curve Surface of the Cylinder will be 2 r c, alſo $\frac{rc}{2}$ will be the Area of a great Circle, (by Sect. IX. of Chap. I. Prob. 1.) and this multiplied by 2 1, makes r 1 c, which is the Solidity of the Cylinder, by Sect. V. of this Chapter. Now, ſince ſ was put equal to 2 r c $=$

the curve Surface of the Cylinder $\frac{ſ\,r}{2}$ (by ſubſtituting ſ for 2 r c) will be alſo = the Solidity of the Cylinder. Now, ſince the Sphere is = $\frac{2}{3}$ of the Cylinder, $\frac{r\,S}{3} = \frac{2 \times ſ\,r}{3 \times 2}$; that is, $\frac{r\,S}{3} = \frac{2\,ſ\,r}{6} = \frac{ſ\,r}{3}$. Wherefore r S = r ſ, that is, dividing by r, S = ſ; or the Surface of the Sphere is equal to the curve Surface of the Cyliuder, but the curve Surface of the Cylinder was 2 r c.

Wherefore to find the Area of the Surface of either Sphere or Cylinder, you muſt multiply the Diameter (= 2 r) by the Circumference of a great Circle of the Sphere, or by the Periphery of the Baſe. From this Notation alſo $\frac{r\,c}{2}$, the Area of a great Circle of the Sphere is plainly $\frac{1}{4}$ of 2 r c, the Surface of the Sphere; that is, the Surface of the Sphere is Quadruple of the Area of the greateſt Circle of it.

Wherefore, to 2 r c, the convex Surface of the Cylinder, add r c, equal to the Area of both its Baſes, you will have 3 r c; which ſhews you, that the Surface of the Cylinder (including its Caſes) is to the Surface of the Sphere as 3 to 2; or that the Sphere is $\frac{2}{3}$ of the circumſcribing Cylinder, in Area as well as Solidity.

Or you may prove the Sphere to be $\frac{2}{5}$ of the Cylinder of the ſame Baſe and Altitude, by Lemma VI. aforegoing, thus:



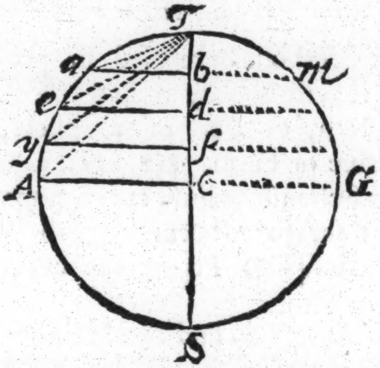
Let A G B repreſent the Hemiſphere, and A I K B half the Cylinder; then, if the Semidiameter G H be divided into ſix equal Parts, and Lines drawn parallel to A B, the Diameter the Squares of the Semichords, ab, cd, ef, &c. will be a Series of Numbers, whoſe greateſt Term AH is a ſquare Number, the other differing by odd Numbers; that is, A H is 36, k l 35, g h 32, e f 27, c d 20, a b 11: But an infinite Series of ſuch Numbers are in Proportion to an infinite Number of Terms equal to the greateſt, as 2 to 3. And becauſe the Hemiſphere is compoſed of an infinite Number of Circles, whoſe Diameters are the Chords of the Semicircle; and the Half-cylinder is compoſed of an infinite Number of Circles, whoſe Diameters are all equal to the Diameter of the Semicircles A B; therefore the Hemiſphere is in Proportion to the Half-cylinder, as 2 to 3; and conſequently, the whole Sphere bears the ſame Proportion to the whole Cylinder.

That the Superficies of every Sphere (or Globe) is equal to four times the Area of its greateſt Circle, is thus proved:

The Solidity of the Sphere is conſtituted of an infinite Number of parallel Circles (as is aforeſaid); conſequently, the Superficies of the Sphere will be compoſed of the Peripheries of thoſe Circles which conſtitute its Solidity.

Note, In the following Demonſtration, ⊙ ſignifies any Circle in general; and if any two Letters be joined to it, thus, ⊙ A B, *&c.* then it denotes the Area of ſuch a Circle as thoſe two Letters repreſent the Radius of.

Let



Let $D = TS$, the Axis of any Sphere; then, according to the Property of a Circle, it

will be	1	$D - Tb \times Tb = \square\, ab$;
that is,	2	$D \times Tb - \square\, Tb = \square\, ab$;
therefore	3	$D \times Tb = \square\, aT$.
For	4	$\square\, AB + \square\, Tb = \square\, aT$ (*Eucl.* 1. 47)
and	5	$D \times Td = \square\, eT$.
	6	$D \times Tf = \square\, yT$.

Hence it is evident, that the Series $\square\, aT$, $\square\, eT$, $\square\, yT$, *&c.* are in the ſame Ratio with Tb, Td, Tf, *&c. viz.* in arithmetical Progreſſion: Whence it follows, that the $\odot\, aT =$ to the Sum of all the Circle's Peripheries between T and b.

And $\odot\, eT =$ the Sum of all the Circles Peripheries between T and d, *&c.*

Conſequently, that the $\odot\, AT =$ the Sum of all the Circles Peripheries, included between T and C; that is, $\odot\, AT =$ the Superficies of the Hemiſphere.

And becauſe $\square\, AC + \square\, TC = \square\, AT$, and $\square\, AC$ is equal to $\square\, TC$; therefore $\odot\, AT = 2 \odot AC$, is the Superficies of the Hemiſphere.

Conſequently, $4 \odot AC$ will be the Superficies of the whole Sphere. Which was to be proved.

Scholium

Scholium.

From the Method here uſed in proving the whole Superficies, it will be eaſy to find the curve Superficies of any Fruſtum, or Part of a Sphere, that is cut off by a right Line or Plain, *viz.* ſuch as the Fruſtum aT m in the laſt Scheme, whoſe curve Superficies is ⊙aT, as above. Therefore (becauſe □ ab + □ Tb = aT) it will be ⊙ ab + ⊙ Tb = the curve Superficies of that Fruſtum.

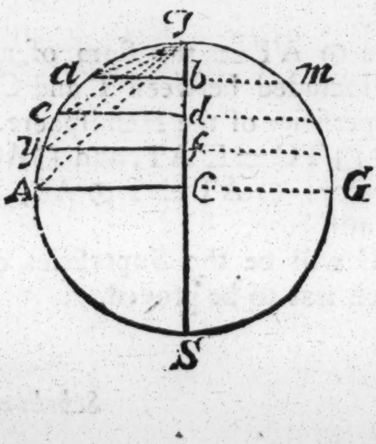
But if the Axis TS, and the Height Tb of the Fruſtum, are given, then it will be TS × Tb = □ aT. as in the third Step above, which gives the Proportion or Theorem following, *viz.*

As the Axis of the Sphere is to the whole Superficies of the Sphere; ſo is the Height of any Fruſtum to its curve Superficies.

To which if there be added the Area of the Fruſtum's Baſe, the Sum will be the whole Superficies of the Fruſtum.

That the Solidity of every Sphere is two-thirds of its circumſcribing Cylinder, *may be thus proved.*

According to the Work above, it appears, that ⊙ ab, ⊙ ed, ⊙ yf, *&c.* do conſtitute the Solidity of the Sphere; and that □ aT, □ eT, □ yT, *&c.* are a Series of Terms in Arithmetical Progreſſion, □ AT being the greateſt Term, and TC the Number of Terms; therefore ⊙ AT × $\frac{1}{2}$ TC = the Sum of all the Series, by Lemma 2.



And becauſe □ aT— □ TB = □ ab. □ eT— □ Td = □ ed. □ yT— □ Tf = □ yf. □ AT— □ TC = □ AC, *&c.*

wherein $\square$ Tb, $\square$ Td, $\square$ Tf, *&c.* are a Series of Squares, whoſe Roots Tb, Td, Tf, are in arithmetical Progreſſion; $\square$ TC being the greateſt Term, and TC the Number of Terms; therefore $\odot$ TC $\times \frac{1}{3}$ TC $=$ the Sum of all the Series, by Lemma 3.

Conſequently, $\odot$ AT $\times \frac{1}{2}$ TC $-$ $\odot$ TC $\times \frac{1}{3}$ TC $=$ the Sum of all the Series $\odot$ ab, $\odot$ ed, $\odot$ yf, *&c.* which conſtitute the Solidity of the Half-ſphere ATG. Put D $=$ 2 TC the Axis of the Sphere; then $\frac{1}{3}$ D $=$ $\frac{1}{2}$ TC, and $\frac{1}{6}$ P $=$ $\frac{1}{3}$ TC. And becauſe $\square$ AT $=$ 2 $\square$ TC, therefore $\odot$ AT $=$ 2 $\odot$ TC $=$ 1.5708 DD; and 1.5708 DD $\times \frac{1}{4}$ D $=$ 0.3927 DDD.

Again, $\odot$ TC $\times \frac{1}{3}$ TC $=$ 0.7854 DD $\times \frac{1}{6}$ D $=$ .1309 DDD, then 0.3927 DDD $-$ 0.1309 DDD $=$ 0.2618 DDD, the Solidity of the Half-ſphere.

Conſequently, 0.2618 DDD $\times$ 2 $=$ .5236 DDD will be the ſolid Content of the whole Sphere, which is equal to $\frac{2}{3}$ of the Cylinder; the Diameter of whoſe Baſe and Height, is $=$ D.

For 0.7854 DDD $=$ the Solidity of the Cylinder by Sect. V. But $\frac{2}{3}$ of 0.7854 DDD $=$ 0.5236 DDD, as before.

Scholium.

From this Demonſtration it will be eaſy to deduce or raiſe Theorems for finding the ſolid Content of any Fruſtum of a Sphere, as Tm in the laſt Figure.

For we there ſuppoſe the Fruſtum a T m to be conſtituted of an infinite Series of Circles, which have the ſame Ratio with all thoſe Circles that conſtitute the Half ſphere.

Therefore it follows, that $\odot$ aT $\times \frac{1}{2}$ Tb: $-$ $\odot$ bT $\times \frac{1}{3}$ Tb will be the Sum of all the Circles intercepted between T and b; conſequently it will be the Solidity of that Fruſtum.

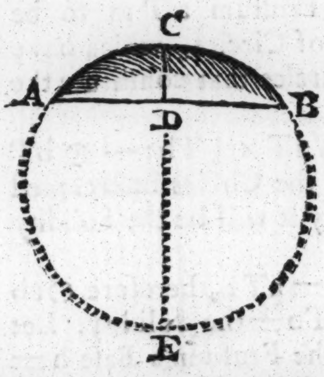
And becauſe $\square$ab $+$ $\square$Tb $=$ aT; therefore $\odot$ab $+$ $\odot$ Tb$\times \frac{1}{2}$ Tb: $-$ $\odot$Tb$\times \frac{1}{5}$ Tb $=$ the Solidity. Let $=$ ab half the Diameter of the Fruſtum's Baſe h $=$ Tb its Height; and S $=$ the Solidity of the Fruſtum.

Then $\odot ab = 3.1416oo$, and $\odot Tb = 3.1416hh$; conſequently, $\frac{3.1416cch + 3.1416hhh}{2} - \frac{3.1416hhh}{3} = S$. Which being reduced, will become $3cch + hhh \times 0.5236 = S$; which is one Theorem for finding the Solidity of the Fruſtum, and may be expreſſed in Words, thus:

If to three times the Square of the Semidiameter of the Fruſtum's Baſe, you add the Square of the Height of the Fruſtum, and multiply the Sum by the Height of the Fruſtum, and that Product multiplied by .5236, the Product will be the ſolid Content.

But if the Axis of the Sphere, and the Height of the Fruſtum, be given; then put $D =$ the Axis, $h =$ the Height of the Fruſtum, and c as before; it will be $D - h \times h = cc$, *viz.* $Dh - hh = cc$. Then will $3Dhh - 2hhh = 3cch + hhh$; conſequently, $3Dhh - 2hhh \times 0.5236 = S$, the Fruſtum's Solidity. Which is another Theorem for finding the Solidity of the Fruſtum, and may be expreſſed in Words, thus:

From three times the Axis ſubtract twice the Height of the Fruſtum, and multiply the Remainder by the Square of the Height, and that Product multiply by .5236, this laſt Product will be the Solidity of the Fruſtum.



Example. Let ABCD be the Fruſtum of a Sphere; ſuppoſe AB (the Diameter of the Fruſtum's Baſe) be 16 Inches, and CD (the Height) 4 Inches; the Solidity is required.

By the firſt Rule.

8
8
64 Square of the Semidiameter AD.
3
192
16 Add the Square of CD.
208
4 Multiply by CD.
832

.5236
832
10472
15708
41888
435.6352

By the ſecond Rule, thus:

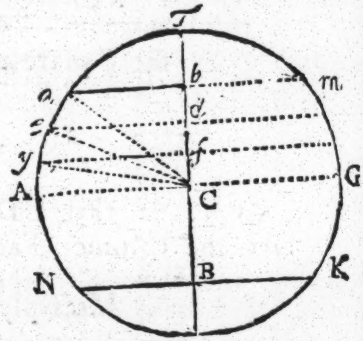
Firſt, by the Rule in Page 113, you will find the Axis of the whole Globe to be 20 Inches.

20 Axis
3
From 60
Subtr. 8 twice CD.
Rem. 52
Mult. 16 Sq. of CD.
312
52
Prod. 832

.5236
832
10472
15708
41888
435.6352 } The Solid Content, the ſame as before.

And if it be required to find the middle Part, amNK, uſually called the middle Zone of a Sphere.

Then, becauſe it is ſuppoſed that $am = NK$, or (which is all one) that $bC = CB$; therefore it is plain, that if twice the Segment, aTm, be taken from the whole Sphere, there will remain the middle Zone amNK.



But becauſe that Work is a little troubleſome, I will here ſhew how to raiſe a Theorem for the doing it,

Firſt, becauſe $AC = yC = eC = aC = TC$, therefore it will be $\square AC - \square Cf = yf$, $\square AC - \square CD = \square ed$, $\square AC - \square Cb = \square ab$, *&c.*

Here becauſe $\square AC$, $\square AC$, $\square AC$, *&c.* are a Series of Equals, and Cb the Number of all the Terms; therefore $\square AC \times Cb =$ the Sum of all that Series. (*per* Lemma I.)

And $\square Cf$, $\square Cd$, $\square Cb$, *&c.* being a Series of Squares, whoſe Roots are in arithmetical Progreſſion, beginning at the Centre, C, *viz.* o, Cf, Cd, Cb, *&c.* wherein the greateſt Term is $\square Cb$, and the Number of Terms is Cb; therefore $\square Cb \times \frac{1}{3} Cb =$ the Sum of all the Series. (*per* Lemma III.)

Conſequently, the $\odot AC \times Cb : - \odot Cb \times \frac{1}{3} Cb =$ the Sum of all the Series $\odot yf$, $\odot ed$, $\odot ab$, *&c.* which do conſtitute the Solidity of the half Zone amAG.

And becauſe $\square AC - \square Cb = \square ab$, therefore $\odot AC - \odot ab = \odot Cb$. Conſequently

$$\odot AC \times Cb : \frac{-\odot AC + \odot ab : \times Cb}{3} = 2\odot AC + \odot ab : \times \tfrac{1}{3} Cb$$

will be the Solidity of the half Zone.

Put $D = AG = 2AC$, $x = am$, and $H = bB = 2Cb$.

Then $\odot AC = .7854DD$, $\odot ab = .7854xx$. And if we turn the common Factor .7854 into a Diviſor 1.27323, and then take the Triple of that Diviſor, *viz.* 3.8197, the Reſult of the precedent Work will produce this following Theorem.

$$\text{Theo.} \left\{ \frac{2DD + xx}{3.8197} : \times H = \right\} \text{The middle Zone amNK.}$$

Which in Words is thus: To twice the Square of the Axis AG, and the Square of the Diameter of the Fruſtum's Baſe (am), and divide the Sum by 3.8197, them multiply the Quotient by the Height or Thickneſs of the middle Zone, and the Product will be the Solidity of the middle Zone required.

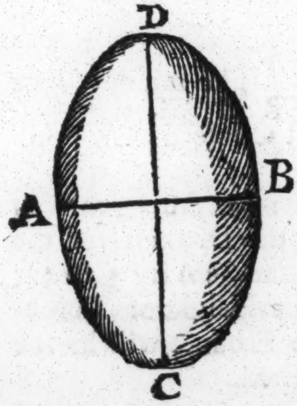
This is ſo plain and eaſy, that it needs no Example.

§ XII. *Of a* SPHEROID.

A Spheroid is a Solid reſembling an Egg. To find the ſolid Content thereof, this is

The RULE.

Multiply the Square of the Diameter of the greateſt Circle by the Length, and that Product multiply again by .5236; this laſt Product will be the Solidity of the Spheroid.



Let AB, the Diameter of the greateſt Circle, be 33 Inches, and CD (the Length) 55 Inches; the Solidity is required.

```
  33
  33
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  99
 99
----
1089
  55
----
5445
5445
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59895
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  59895
  .5236
-------
 359370
179685
119790
299475
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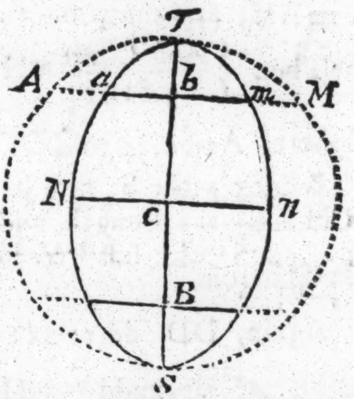
31361.0220 The Solidity.
```

Demonſtration. Every Spheroid is equal to $\frac{2}{3}$ of a Cylinder, whoſe Baſe is equal to the greateſt Circle of the Spheroid, and its Height equal to the Length of the Spheroid.



Suppoſe the Figure NTnSN in the annexed Scheme, to repreſent a Spheroid, formed by the Rotation of the Semi-Ellipſis TNS, about its tranſverſe Axis TS.

Let D = TS, the Length of the Spheroid, and the Axis of the circumſcribing Sphere; and d = Nn the Diameter of the greateſt Circle of the Spheroid,

Then, becauſe □ TC : □ NC : : □ Ab : □ ab, by *Sect*. XV. *Step*. 3. *Page* 125.

Therefore it will be, DD : dd : : □ Ab : □ ab.

But the Sum of an infinite Series of ſuch Circles as ⊙ Ab (whoſe Diameters are Chords) do conſtitute the Solidity of the Sphere. (By *Sect*. XI.)

And the Sum of an infinite Series of ſuch Circles as ⊙ ab (*viz.* whoſe Diameters are Ordinates of the Ellipſis) do conſtitute the Solidity of the Spheroid.

Therefore, DD : dd : : 0.5236 DDD : 0.5236 Ddd = the Solidity of the Spheroid. (*Eucl.* 5. 12.)

But 0.5236 Ddd = $\frac{2}{3}$ of the Cylinder, whoſe Diameter is = d, and Height = D. (By *Sect*. V.)

Now, from this Proportion, between the Sphere and its inſcribed Spheroid, it will be very eaſy to deduce Theorems for finding the ſolid Content, either of the Fruſtum or middle Zone of any Spheroid; having the ſame Height with that of the Sphere; for,

As the Solidity of the whole Sphere is to the Solidity of the whole Spheroid; ſo is any Part of the Sphere to the like Part of the Spheroid.

As for Inſtance: Suppoſe it was required to find the middle Zone of any Spheroid.

Let $D=TS$, and $d=Nn$, as above; and $H=bB$, $x=AM$, and $c=am$.

Then $\left\{\frac{2DD+xx}{3.8197}\right.\times H=$ the middle Zone of the Sphere. And $0.5236\,DDD : 0.5236\,ddD :: \frac{2DD+xx}{3.8197}\times H : \frac{2ddH}{3.8197}+\frac{xxddH}{3.8197DD}=$ the middle Zone of the Spheroid.

Again, $DD : dd :: xx : cc$. Therefore $\frac{xxdd}{DD}=cc$. Conſequently, $\frac{xxdd}{DD}\times\frac{H}{3.8197}=\frac{cc}{3.8197}\times H$ Which being taken inſtead of $\frac{xxddH}{3.8197DD}$, there will ariſe the following Theorem $\left\{\frac{2dd+cc}{3.8197}\right. : \times H=$ the middle Zone of the Spheroid.

Note, That $3.8197=1.2732\times 3$. See *Page* 102.

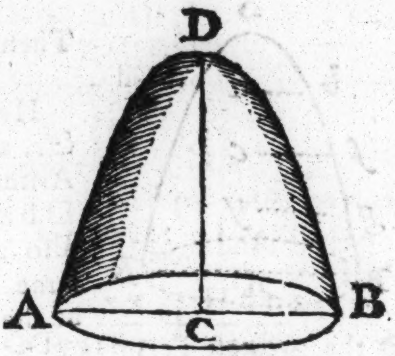
§ XIII. *Of a Parabolic* CONOID.

A Parabolic Conoid is ſomething like a half Spheroid having its Sides ſomewhat ſtraighter. It is generated by ſuppoſing a Semi-parabola turned about its Axis. To find the ſolid Content thereof, this is

The RULE.

Multiply the Square of the Diameter of its Baſe by .7854, and multiply that Product by half the Height, that laſt Product ſhall be the ſolid Content.

Let ABCD be a Parabolic Conoid, the Diameter of whoſe Baſe is 36 Inches, and its Height CD 33 Inches; the Solidity is required.



36
36
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216
101
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1296

.7854
1296
——
47124
70686
15708
7854
——
1017.8784

1017.8784
33
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30536352
30536352
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2)33589.9872
——
16794.9936

1728)16794.99|36(9.71 Feet the Content.
15552
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12429
12096
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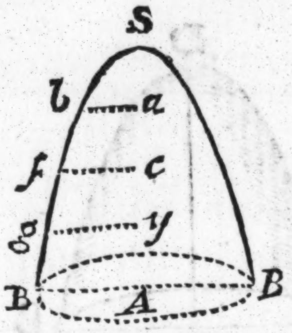
3339
1728
——
1611

Demonſtration. The Parabolic Conoid is conſtituted of an infinite Number of Circles, whoſe Diameters are the Ordinates of the Parabola. Now according to the Property of every Parabola, it will be, $SA : AB :: AB : \frac{\square AB}{SA} = L.$ the *Latus Rectum.*

Then



$$\text{Then}\begin{cases} Sa\times L = \square\, ba, \\ Se\times L = \square\, fe, \\ Sy\times L = \square\, gy, \text{ \&c.} \end{cases}$$

Here $Sa\times L$, $Se\times L$, $Sy\times L$, &c. are a Series of Terms in Arithmet. Progreſſ. Therefore $\square\, ba$, $\square\, fe$, $\square\, gy$, &c. are alſo a Series of Terms in the ſame Progreſſion, beginning at the Point S, wherein $\square\, AB$ is the greateſt Term, and SA the Number of all the Terms. Therefore $\square\, AB\times\frac{1}{2}SA =$ the Sum of all the Series. (By Lemma 2.)

Conſequently, $\odot\, AB\times\frac{1}{2}SA =$ the Sum of all the Series of $\odot\, ba$, $\odot\, fe$, $\odot\, gy$, &c. which do conſtitute the Solidity of the Conoid.

Put $D = 2\,AB$, and $H = SA$.

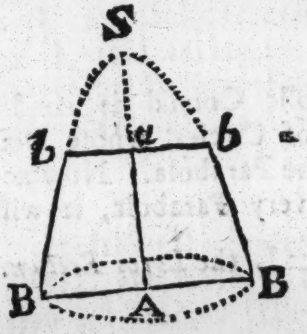
Then $.7854\,DD\times\frac{1}{2}H = .3927\,DDH$ will be the ſolid Content of the Conoid; which is juſt half the Cylinder, whoſe Baſe is $= D$, and Height $= H$.

This being rightly underſtood, it will be eaſy to raſe a Theorem for finding the lower Fruſtum of any Parabolic Conoid.

For, ſuppoſing $h = aA$, the Height of the Fruſtum, and $p = Sa$, the Height of the Part bSb cut off, and $h + p = SA$, the Height of the whole Conoid.

Conſequently, $\dfrac{\odot\, AB\times H + \odot\, AB\times p}{2} =$ the Solidity of the whole Conoid.

And $\dfrac{\odot\, ba\times p}{2} =$ the Solidity of the Part cut off.



Therefore

Therefore	1	$\frac{\odot AB\times h+\odot AB\times p-\odot ba\times p}{2}$ The Solidity of the Fruſtum.
But	2	$h+p ; \square AB :: p : \square ba$
Conſequ.	3	$h+p : \odot AB :: p : \odot ba$
$3 \because$	4	$\odot AB\times p := \odot ba\times ph+\odot ba\times p$
$4-\odot ba\times p$	5	$\odot AB\times p := \odot ba\times h+\odot ba\times h$
1×2	6	$\odot AB\times h :+\odot AB\times p :-\odot ba\times p=2F.$
$6-5$	7	$\odot AB\times h=2F :-\odot ba\times h.$
$7+\odot ba\times h$	8	$\odot AB\times h :+\odot ba\times h=2F.$
$8\div 2$	9	$\frac{\odot AB+\odot ba}{2} :\times h=F$, the Fruſtum's Solidity.

Let $D=2AB$, as before, and $d=2ba$, the Diameter of the Part cut off; then we ſhall have the following Theorem.

$0.3927\,DD+0.3927\,dd :\times h=$ the Solidity of the Fruſtum required: Which in Words is thus:

Multiply the Sum of the Squares of the greater and leſſer Diameters by .3927, and that Product by the Height of the Fruſtum, the laſt Product ſhall be the ſolid Content.

§ XIV. *Of a Parabolic* SPINDLE.

IF an acute Parabola be ſuppoſed to be moved about its greateſt Ordinate, it will form a Solid, called a Parabolic Spindle. To find the ſolid Content, this is

The RULE.

Multiply the Square of the Diameter of its greateſt Circle by .41888, being $\frac{8}{5}$ of .7854) and that Product by its Length; that laſt Product is the ſolid Content.

Let ABCD be a Parabolic Spindle, whoſe greateſt Diameter CD is 36 Inches, and its Length AB 99 Inches; the Solidity is required.

36=CD
36

216
108

1296 Square

.41888
1296

251328
376992
83776
41888

542.86848
99

488581632
488581632

1728)53743.97952(31.10184
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1903
1759
3179
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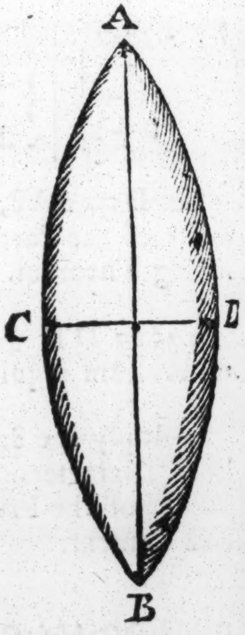
6912

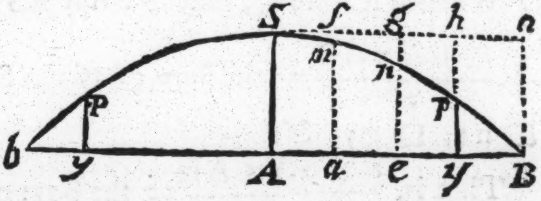
. . . .



The Solid Content is 31.10184 Feet.

Demonſtration. A Parabolic Spindle is conſtituted of an infinite Series of Circles, whoſe Diameters are all parallel to the Axis of the Parabola, as ⊙ ma, ⊙ ne, ⊙ py, *&c.*

Let us ſuppoſe the Line S d parallel to A B, *&c.* Then it hath already been proved, that the Lines f m, g n, h p, *&c.* are a Series of Squares, whoſe Roots are in arithmetical Progreſſion, conſequently their Squares, *viz.* □ fm, □ gn, □ hp, *&c.* will be a Series of Biquadrats, whoſe Roots will be in arithmetical Progreſſion: Which being premiſed, we may proceed thus:



Firſt { 1 | S A — f m = m a.
2 | S A — g n = n e.
3 | S A — h p = p y.
1 ⊚ 2 | 4 | □ S A — 2 S A × fm + □ fm = □ ma.
2 ⊚ 2 | 5 | □ S A — 2 S A × gn + □ gn = □ ne.
3 ⊚ 2 | 6 | □ S A — 2 S A × hp + □ hp = □ py, *&c.*

1. In theſe Equations, the □ S A, □ S A, □ S A, being a Series of Equals, and A B the Number of all the Terms; therefore it will be □ S A × A B = the Sum of the Series. (By Lemma I.)

2. Becauſe f m, g n, h p, *&c.* are a Series of Squares, wherein S A is the greateſt Term, and AB the Number of all the Terms;

Therefore $\frac{2SA \times SA \times AB}{3} = \frac{2\square SA \times AB}{3}$ will be the Sum of all the Series. (By Lemma III.)

3. And the $\square\, fm$, $\square\, gn$, $\square\, hp$, *&c.* will be a Series of Terms in the Ratio of Biquadrats as above, $\square\, SA$ being the greateſt Term, and AB the Number of all the Terms. Therefore it will be $\frac{\square\, SA \times AB}{5}$ = the Sum of all the Series. (By Lemma V.)

Whence it follows, that $\square\, SA \times AB - \frac{2\,\square\, SA \times AB}{3} + \frac{\square\, SA \times AB}{5}$ = the Sum of all the Series of $\square\, ma$, $\square\, ne$, $\square\, py$, *&c.*

That is, $\frac{8\,\square\, SA \times AB}{15}$ = the Sum of all the Series $\square\, ma$, $\square\, ne$, $\square\, py$, *&c.* Conſequently, $\frac{8 \odot SA \times AB}{15}$ = the Sum of all the Series of Circles, $\odot\, ma$, $\odot\, ne$, $\odot\, py$, *&c.* which conſtitute the Solidity of half the Spindle, *viz.* of SAB.

Therefore putting $D = 2\,SA$, and $H = 2\,AB$, it will be $0.41888\, DDH$ = the Solidity of the whole Parabolic Spindle bSB, being $\frac{8}{15}$ of $0.7854\, DDH$, the Solidity of its circumſcribing Cylinder.

From hence we may alſo raiſe a Theorem for finding the Fruſtum, $SApy$, of the laſt Figure.

For $\odot\, SA$ being the greateſt Term, $\odot\, py$ the leaſt Term, and Ay the Number of all the Terms or Circles included between A and y:

There-fore 1 $\square SA - \frac{2SA \times hp}{3} + \frac{\square hp}{5} : \times Ay =$ z the Sum of all the Series, □ SA, □ ma, □ en, □ py,

1 × 3 2 $3\square SA - 2SA \times hp + \frac{3\square hp}{5} : \times Ay = 3z$

z — Ay 3 $3\square SA - 2SA \times hp + \frac{3\square hp}{5} = \frac{3z}{Ay}$

But 4 $\square SA - 2SA \times hp = \square py - \square hp$, *per St.* 6.

3—4 5 $2\square SA + \frac{\square 3hp}{5} = \frac{3z}{Ay} - \square py + \square hp$.

∓ &c. 6 $2\square SA + \square py - \frac{2}{5}\square hp = \frac{3z}{Ay}$

Conſeq. 7 $2\odot SA + \odot py - \frac{2}{5}\odot hp : \times \frac{1}{3}Ay = z$ the Sum of all the Series of ⊙ SA, ⊙ ma, ⊙ ne, ⊙ py, which do conſtitute the Solidity of the Fruſtum SApy. Therefore putting $D = 2SA$, as before, $C = 2py$, $x = 2hp$, and $H = Ay$; it will be $1.5708\,DD + .7854\,CC - .31416\,xx : \times \frac{1}{3}H =$ the Fruſtum SApy. And if we make $L = 2H$, then $1.5708\,DD + .7854\,CC - .31416\,xx : \times \frac{1}{3}L =$ the Double of that Fruſtum, being the middle Zone. Which in Words is thus:

Multiply the Square of the greateſt Diameter by 1.5708, and multiply the Square of the leſſer Diameter by .7854, and multiply the Square of the Difference of the Diameters by .31416; from the Sum of the two former Products ſubtract the latter Product, and multiply the Remainder by one third Part of the Length, and that Product will be the Solidity of the middle Zone required.

CHAP. III.

Of the Meaſuring the Works of the ſeveral Artificers relating to Building; and what Methods and Cuſtoms are obſerved therein.

§ I. *Of* CARPENTERS *Work.*

THE Carpenters Work which are meaſurable, are Flooring, Partitioning, and Roofing; all which are meaſured by the Square of 10 Feet long, and 10 Feet broad; ſo that one Square contains 100 Square Feet.

1. *Of Flooring.*

If a Floor be 57 Feet 3 Inches long, and 28 Feet 6 Inches broad; How many Squares of Flooring are there in that Room?

Multiply 57 Feet 3 Inches by 28 Feet 6 Inches, and the Product is 1631 Feet, *&c.* which divide by 100 (this is done by cutting from the Product two Figures towards the Right-hand with a Daſh of the Pen); and the remaining Figures are the Quotient, and the Figures cut off are Feet: Thus, 1631 divided by 100, by cutting off 31 from the Right-hand thereof, the Quotient is 16 Squares, and the 31 cut off is 31 Feet.

See the Work, both by Decimals, and alſo by Feet and Inches.

57.25	F.	I.	
28.5	57	3	
———	28	6	
28625	———	———	
45800	456		
11450	114		
———	28	7	6
16|31.625	7	0	0
	———	———	———
	16|31	7	6

Facit 16 Squares and 31 Feet.

Note, That 5 is the Decimal for half of any thing, .25 is the Decimal for a Quarter, and .125 is the Decimal for half a Quarter; ſo in the laſt Example, .25 is the Decimal of 3 Inches, becauſe 3 Inches is a Quarter of a Foot; and .5 is the Decimal of 6 Inches, becauſe 6 Inches is half a Foot.

Example 2. Let a Floor be 53 Feet 6 Inches long, and 47 Feet 9 Inches broad; How many Squares are contained in that Floor?

47.75	F.	I.	
53.5	53	6	
———	47	9	
23875	———	———	
14325	371		
23875	212		
———	26	9	
25|54.625	13	4	6
	23	6	
	———	———	———
	25|54	7	6

Facit 25 Squares and 54 Feet

By

By Scale and Compaſſes.

In the firſt Example, extend the Compaſſes from 1 to 28.5, that Extent will reach from 57.25 to 16 Squares and near a third Part.

In the ſecond Example, extend the Compaſſes from 1 to 47.5, that Extent will reach from 53 5 to 25 Squares and above an Half.

2. *Of Partitioning.*

Example 1. If a Partition between Rooms be in Length 82 Feet 6 Inches, and in Height 12 Feet 3 Inches; How many Squares are contained therein?

The Length and Breadth being multiplied together, the Product is 1010.625; which divide by 100, (as before is ſhewed) and the Anſwer is 10 Squares 10 Feet; the Inches or Parts, in theſe Caſes, are of no Value.

12.25	F.	I.	
82.5	82	6	
	12	3	
6125			
2450	990	0	
9800	20	7	6
10|10.625	10|10	7	6

Facit 10 Squares 10 Feet.

Example 2. If a Partition between Rooms be in Length 91 Feet 9 Inches, and in Breadth 11 Feet 3 Inches; How many Squares are contained therein?

The Length and Breadth being multiplied together, the Product is 1032 Feet; which divided by 100, the Anſwer will be 10 Squares and 32 Feet.

91.75

91.75
11.25

45875
18350
9175
9175

10|32.1875

F. I.
61 9
11 3

1009 3
22 11 3

10|32 2 3

61 - 9
11 - 3
671
5 - 6
2 - 9
15 - 3

3. *Of Roofing.*

It is a Rule amongſt Workmen, that the Flat of any Houſe, and half the Flat thereof, taken within the Walls, is equal to the Meaſure of the Roof of the ſame Houſe; but this is when the Roof is true pitched: For if the Roof be more flat or ſteep than the true Pitch, it will meaſure to more or leſs accordingly.

Example 1. If a Houſe within the Walls be 44 Feet 6 Inches long, and 18 Feet 3 Inches broad; How many Squares of Roofing will cover that Houſe?

Multiply the Length and Breadth together, and the Product is 812 Feet, the Flat; the half thereof is 406 Feet; which added to the Flat, the Sum is 1218 Feet; which divided by 100, the Anſwer is 12 Squares and 18 Feet.

61 - 9
11 - 3
61
61
15 - 5 - 3
5 - 6
2 - 9 - 0
784 . 8 . 3

18.25	F. I.
44 5	44 6
	18 3
9125	
7300	352
7300	44
	11 1 6
Flat 812.125	9 0 0
Half 406	
	The Flat 812 1 6
12|18	The Half 406
	Sum 12|18

Facit 12 Squares 18 Feet.

By Scale and Compaſſes.

In the firſt Example of Partitioning, extend the Compaſſes from 1 to 12.25, that Extent will reach from 82.5 to 10 Squares and One tenth.

In the ſecond Example, extend the Compaſſes from 1 to 11.25, that Extent will reach from 91.75 to 10 Squares, and a little leſs than a third Part.

In the Example of Roofing, extend the Compaſſes from 1 to 18.25, that Extent will reach from 44.5 to 812 the Flat; to which add the Half thereof, and the Sum is 12.18, which is 12 Squares 18 Feet, as above.

There are other Works about a Building, done by the Carpenter, which are meaſured by the Foot, running Meaſure, that is, by the Number of Feet in Length only; as Cornices, Doors and Caſes, Window-frames, Guttering, Lintels, Sommers, Skirt-boards, *&c.*

Note 1. In the Meaſuring of Flooring, after you have meaſured the whole Floor, you muſt deduct out of it the Well-holes for the Stairs and Chimneys; and in Partitioning, for the Doors Windows, *&c.* except (by Agreement) they are to be included.

Note 2. In meaſuring of Roofing, ſeldom any Reductions are made for the Holes for the Chimney-ſhafts, the Vacancies for Lutheren-lights and Sky-lights; for they are more Trouble to the Workman, than the Stuff which would cover them is worth.

§ II. *Of* BRICKLAYERS *Work*.

THE principal is Tiling, Walling, and Chimney-work.

1. *Of Tiling*.

Tiling is meaſured by the Square of 100 Feet, as Flooring, Partitioning, and Roofing were, in the Carpenters Work; ſo that between the Roofing and Tileing, the Difference will not be much; yet the Tiling will be the moſt; for the Bricklayers ſometimes will require to have double Meaſure for Hips and Vallies. When Gutters are allowed double Meaſure, the way is to meaſure the Length along the Ridge-tile, and by that means the Meaſure of the Gutters becomes double; it is uſual alſo to allow double Meaſure at the Eaves, ſo much as the Projector is over the Plate, which is commonly about 18 or 20 Inches.

Example 1. There is a Roof covered with Tiles, whoſe Depth on both Sides (with the uſual Allowance at the Eaves) is 37 Feet 3 Inches, and the Length 45 Feet; I demand how many Squares of Tiling are contained therein?

F. I.
37 3
45 0

185
148
11 3

16|76 3

37.25
45

18625
14900

16|762.5

Anſwer, 16 Squares 76 Feet.

Example 2. There is a Roof covered with Tiles, whoſe Depth on both Sides (with the Allowance at the Eaves) is 35 Feet 9 Inches, and the Length 43 Feet 6 Inches; I demand how many Squares of Tileing are in the Roof?

F. I.
43 6
35 9

215
129
21 9
10 10 6
17 6

15|55 1 6

35.75
43.5

17875
10725
14300

15|55.125

Here the Length and Depth being multiplied together, the Product is 1555 Feet; which divided by 100 (as before is taught) the Anſwer is 15 Squares and 55 Feet.

By Scale and Compaſſes.

In the firſt Example, extend the Compaſſes from 1 to 37.25, that Extent will reach from 45 to 16 Squares and a little above three Quarters of a Square.

In the ſecond Example extend the Compaſſes from 1 to 35.75, that Extent will reach from 43.5 to 15 Squares and 55 Feet, that is, a little above a Half-ſquare.

2. *Of Walling.*

Bricklayers commonly meaſure their Work by the Rod Square of 16 Feet and a half; ſo that one Rod in Length and 1 in Breadth, contain 272.25 Square Feet; for 16.5, multiplied in itſelf, produces 272.25 Square Feet. But in ſome Places the Cuſtom is to allow 18 Feet to the Rod; that is 324 Square Feet. And in ſome Places the uſual Way is, to meaſure by the Rod of 21 Feet long and 3 Feet high, that is, 63 Square Feet; and here they never regard the Thickneſs of the Wall, but the uſual Way is to moderate the Price according to the Thickneſs.

When you meaſure a Piece of Brick-work, the firſt thing is to inquire by which of thoſe Ways it muſt be meaſured; then, having multiplied the Length and Breadth in Feet together, divide the Product by the proper Diviſor, either for Rods or Roods, and the Quotient is Square Rods, or Square Roods, accordingly.

But commonly Brick-walls, that are meaſured by the Rod, are to be reduced to a Standard Thickneſs, *viz.* of a Brick and a half thick (if it be not agreed on the contrary); and to reduce a Wall to Standard-thickneſs, this is

The RULE.

Multiply the Number of ſuperficial Feet that are found to be contained in any Wall, by the Number of Half-bricks which that Wall is in Thickneſs; one third Part of that Product ſhall be the Content thereof in Feet, reduced to the Standard-thickneſs of one Brick and a half.

Example 1. If a Wall be 72 Feet 6 Inches long, and 19 Feet 3 Inches high, and 5 Bricks and a half thick; How many Rods of Brick-work are contained therein, when reduced to the Standard?

```
                 19.25 Height.
                  72.5 Length.
                 -----
                  9625
                 3850
               13475
               --------
               1395.625
                     11
               --------
            3)15351.875
            -----------
   272.25)5117.291(18 Rods
          ---------
            239479
            ------
     68.06)21679(3 Quarters of a Rod.
            -----
            12.61
```

Anſwer, 18 Rods 3 Quarters 12 Feet.

```
     F.   I.
     72   6
     19   3
     ———————
    648
    72
     18   1  6
      9   6  0
    ————————————
   1395   7  6
     11
   ——————
 3)15345
   ——————
272)5115(18 Rods.
   ——————
    2395
   ——————
 68)219(3 Quarters of a Rod.
   ————
     15
```

272)5115(

2997

21

Note, That 68.06 is one fourth Part of 272.25

Note alſo, That in reducing of Feet into Rods, hey uſually reject the odd Parts, and divide only by 72, as is done in the ſecond Way of the laſt Example; o the Anſwer, by that ſecond Way, is 18 Rods 3 Quarters and 15 Feet, more by about 2½ Feet than by he firſt Way, where it is done decimally; a Thing ery inſignificant.

Example 2. If a Wall be 245 Feet 9 Inches long, and 16 Feet 6 Inches high, and two Bricks and a half thick; I demand how many Rods of Brick-work are contained therein, when reduced to Standard-thickneſs?

245.75
16.5

122875
147450
24575

4054.875
5

3)20270

272)6756(24 Rods.

1316

68)228(3 Quarters of a Rod.

24

Anſwer, 24 Rods 3 Quarters 24 Feet.

F.	I.	
245	9	
16	6	
1470		
245		
122	10	6
12	0	0
4054	10	6 Anſwer in Feet.

Before I ſhew how to work the two laſt Examples by Scale and Compaſſes, I will ſhew how to find proper Diviſors to facilitate the Operation; becauſe it would be too intricate and tedious to perform by Scale and Compaſſes, according to the Rule above taught.

To find proper Diviſors.

Divide 3 (the Number of Half-bricks in 1½) by the Number of Half-bricks in the Thickneſs, the Quotient will be a Diviſor, which will give the Anſwer in Feet. But if you would have a Diviſor to bring the Anſwer in Rods at once, then multiply 272.25 by the Diviſor found for Feet, and the Product will be a Diviſor, which will give the Anſwer in Rods.

Example. Let it be required to find a Diviſor proper to reduce a Wall of three Bricks thick.

Divide 3 by 6 (the Half-bricks in the Thickneſs) and the Quotient is .5, which is a Diviſor that will give the Anſwer in Feet. Then multiply 272.25 by .5, and the Product is 136.125, the Diviſor, which will give the Anſwer in Rods; that is, as 136.125 is to the Length of the Wall, ſo is the Height to the Content in Rods. Or, as .5 is to the Length, ſo is the Height to the Content in Feet.

After the ſame manner you may find Diviſors for any other Thickneſs, which you will find to be as expreſs'd in the following little Table.

The Thickneſs of the Wall.	Diviſors for the Anſwer in Feet.	Diviſors for bringing the Anſwer in Rods.
1 Brick thick	1.5	408.375
1 ½ Brick thick	1.	272.25
2 Bricks thick	.75	204.1875
2 ½ Bricks thick	.6	163.35
3 Bricks thick	.5	136.125
3 ½ Bricks thick	.4285	116.659
4 Bricks thick	.375	102.0937

Let the ſecond Example, aforegoing, be wrought by Scale and Compaſſes, where the Length is 245.75, the Height 16.5, and the Thickneſs 2½ Bricks.

Extend the Compaſſes from 163.35, (the tabular Number againſt 2½ Bricks) to 245.75; that Extent will reach from 16.5 to 24 Rods and 8 Tenths.

Again, if the Length be 75 Feet 6 Inches, and the Height 18 Feet 9 Inches, at 3½ Bricks thick; How many Rods are contained therein?

Extend the Compaſſes from 116.678 (the tabular Number) to 18.75, that Extent will reach from 75.5 to 12.13; that is 12 Rods and a little above half a Quarter.

It will be very proper and commodious, for ſuch as have frequent Occaſion to meaſure Brick-work, to have in the Line of Numbers little Braſs Centre-pins at each of the Numbers in the third Column of the above little Table, with a Figure to denote the Thickneſs of the Wall.

If a Wall be 104 Feet 9 Inches long, and 17 Feet 3 Inches high, how many Rods are contained therein?

	F.	I.	
104.75	104	9	
17.25	17	3	
52375	728		
20950	104		
73325	26	2	3
10475	12	9	0
63)1806.9375(28	1806	11	3

126

Anſwer, 28 Rods 42 Feet.

546

504

42

Note, That ſuch as dig Cellars, do many times do them by the Floor, 16 Feet ſquare, and a Foot deep, being a Floor of Earth, that is, 324 ſolid Feet.

3. *Of Chimnies.*

If you are to meaſure a Chimney ſtanding alone by itſelf, without any Party-wall being adjoined, then girt it about for the Length, and the Height of the Story is the Breadth; the Thickneſs muſt be the ſame as the Jambs are of, provided that the Chimney be wrought upright from the Mantle-tree to the Cieling, not deducting any thing for the Vacancy between the Floor (or Hearth) and the Mantle tree, becauſe of the Gatherings of the Breaſt and Wings, to make room for the Hearth in the next Story.

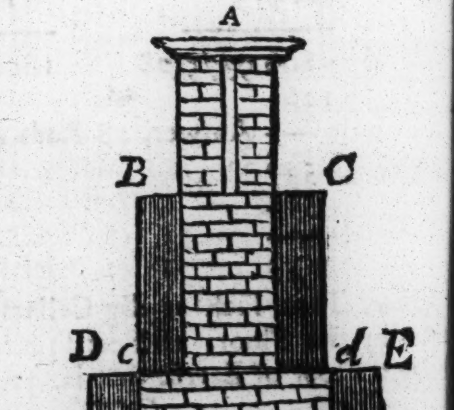
If the Chimney-back be a Party-wall, and the Wall be meaſured by itſelf, then you muſt meaſure the Depth of the two Jambs, and the Length of the Breaſt for a Length, and the Height of the Story the Breadth, at the ſame Thickneſs your Jambs were of.

When you meaſure Chimney ſhafts, girt them with a Line round about the leaſt Place of them, for the Length, and the Height ſhall be your Breadth: And if they be four Inch Work, then you muſt ſet down your Thickneſs at one Brick-work; but if they be wrought 9 Inches thick, (as ſometimes they are, when they ſtand high and alone above the Roof) then you muſt account your Thickneſs 1½ Brick, in conſideration of Withs and Pargetting, and Trouble in Scaffolding.

It is cuſtomary, in moſt Places, to allow double Meaſure for Chimneys.

Example. Suppoſe this Figure, ABCDEFGHIK, to be a Chimney that hath a double Tunnel towards the Top, and a double Shaft, and is to be meaſur'd according to double Meaſure.

First, I begin with the Breaſt-wall I L, and the two Angles L K and HI, which together are 18 Feet 9 Inches; then take the Height of the Square H F, 12 Feet 6 Inches, which multiply'd together, produce 234 Feet 4 Inches 6 Parts, for the Content of the Figure FGHK.



For the Square DaEb, the Length of the Breaſt-wall and two Angles, is 14 Feet 6 Inches, and the Height Da 9 Feet; which multiply'd together, make 130 Feet 6 Inches, for the Content of the Part DaEb.

Then the Height of the next Square 7 Feet, and the Length of the Breaſt-wall and two Angles is 10 Feet 3 Inches; which multiply'd together, produceth 71 Feet 9 Inches, for the Content of the Square BcCd.

The Compaſs of the Chimney-ſhafts is 13 Feet 9 Inches, and the Height 6 Feet 6 Inches; which multiply'd together, make 89 Feet 4 Inches 6 Parts, the Content of the Shafts.

The Depth of the middle Fetter, that parts the Funnels, is 12 Feet, and its Wideneſs 1 Foot 3 Inches; which multiply'd together, make 15 Feet, the Content thereof.

The Work.

	F.	I			
	18	9			18.75
	12	6			12.5
	225	0			9375
	9	4	6		3750
					1875
FGHK	234	4	6	FGHK	234.375
	F.	I.			
	14	6			14.5
	9	0			9
DaEb	130	6		DaEb	130.5
	F.	I.			
	10	3			10.25
	7	0			7
BcCd	71	9		BcCd	71.75

F. I,
13 9
6 6
82 6
6 10 6
The Shaft 89 4 6

13.75
6.5
6875
8250
The Shaft 89.375

F. I.
1 3
12 0
The Fetter 15 0

1.25
12
The Fetter 15.00

272)1082(3 Rods
68) 266(3 Quarters.
Rem. 62 Feet.

	F.	I.	P.
FGHK	234	4	6
DaEb	130	6	0
BcCd	71	9	0
The Shaft	89	4	6
The Fetter	15	0	0
The Sum	541	0	0
The Double	1082	0	0

Having added the five Products together, and doubled the Sum, that double Sum is the Content of the Chimney in Feet, according to double or cuſtomary Meaſure; which Feet muſt be reduc'd to Rods, as was ſhew'd before.

So the Feet in the foregoing Example being reduc'd to Rods, (the Thickneſs being ſuppoſed 1½ Bricks) i makes 3 Rods 3 Quarters and 62 Feet; that is, 4 Rod wanting 6 Feet.

This is all the Meaſure that can be allow'd, whe the Chimney ſtands in a Gavel or Side-wall; in which Caſe the Back of the Chimney (here not meaſur'd) is accounted as Part of the Gavel; but if the Chimneys ſtand by themſelves, as all Stacks of Chimneys in great Buildings do, in ſuch Caſe it is all Chimney-work, and therefore ought to be meaſured double on all Sides.

§ III. *Of* PLAISTERERS *Work.*

THE Plaiſterers Work are principally of two Kinds, namely, 1. Works lath'd or plaiſter'd, which they call Ceiling. 2. Works render'd; which are of two Kinds, *viz.* upon Brick-walls, or between Quarters, in the Partitions between Rooms; all which are meaſured by the Yard ſquare, or Square of 3 Feet, which is 9 Feet.

1. *Of Ceiling.*

If a Ceiling be 59 Feet 9 Inches long, and 24 Feet 6 Inches broad, how many Yards doth that Ceiling contain?

Multiply 59 Feet 9 Inches by 24 Feet 6 Inches, and the Product is 1463 Feet 10 Inches 6 Parts; which divided by 9, the Quotient is 162 Yards 5 Feet.

F.	I.		59.75
59	9		24.5
24	6		
			29875
236			23900
118			11950
29	10	6	
18	0	0	9)1463.875
1463	10	6 Anſwer	162.65

By

By Scale and Compaſſes.

Extend the Compaſſes from 9 to 59 Feet 9 Inches, that Extent will reach from 24 Feet 6 Inches to 16.25 Yards.

2. *Of Rendering.*

Example. If the Partitions between Rooms be 141 Feet 6 Inches about, and 11 Feet 3 Inches high; How many Yards are in thoſe Partitions?

Multiply 141 Feet 6 Inches by 11 Feet 3 Inches, and the Product is 1591 Feet 10 Inches 6 Parts; which divided by 9, gives 176 Yards 7 Feet, the Anſwer.

F.	I.		
141	6		141.5
11	3		11.25
1556	6		7075
35	4	6	2830
			1415
9)1591	10	6	1415
Anſwer 176	7		9)1591.875
			176.87

Anſwer, 176.87 Yards.

Extend the Compaſſes from 9 to 141.5, that Extent will reach from 11.25 to 176.87 Yards.

Note 1. If there be any Doors, Windows, or the like, in your Partitioning, you muſt make Deductions for them.

Note 2. When you meaſure Rendering upon Brick-walls, you are to make no Deductions; but when you meaſure Rendering between Quarters, you may very well deduct one-fifth Part for the Quarters, Braces, and Entertices.

Note 3. That Whiting and Colouring are both meaſured by the Yard, as Cieling and Rendering were; and as in Rendering between Quarters, you deduct one-fifth Part, ſo in Whiting and Colouring you muſt add one-fourth, or one-fifth Part at leaſt.

* * *

§ IV. *Of* JOYNERS *Work.*

JOyners do meaſure their Work by the Yard ſquare; but in taking their Dimenſions, they differ from ſome others; for they have a Cuſtom, and ſay, *We ought to meaſure where our Plane touches:* Wherefore, in taking the Height of any Room, where there is a Cornice about, and ſwelling Panels and Mouldings, they, with a String, begin at the Top, and girt over all the Mouldings; which will make the Room to meaſure much higher than it is: Then for meaſuring about the Room, they only take it as is is upon the Floor.

Example 1. If a Room of Wainſcot (being girt downwards over the Mouldings) be 15 Feet 9 Inches high, and 126 Feet 3 Inches in Compaſs; How many Yards doth that Room contain?

Multiply the Compaſs by the Height, and the Product is 1988 Feet 5 Inches 3 Parts; which divided by 9, gives 220 Yards and 8 Feet, the Anſwer.

9)1591(176-5-10.5
66(5(

F.	I.	
126	3	
15	9	
630		
126		
63	1	6
31	6	9
3	9	0
9)1988	5	3
Anſwer 220	8	

126.25
15.75
63125
88375
63125
12625
9)1988.4375
220 8

Facit 220 Yards 8 Feet.

Example 2. If a Room of Wainſcot be 16 Feet 3 Inches high, (being girt over the Mouldings) and the Compaſs of the Room 137 Feet 6 Inches; How many Yards are contain'd therein?

Multiply 137 Feet 6 Inches by 16 Feet 3 Inches, and the Product is 2234 Feet 4 Inches 6 Parts; which divided by 9, the Quotient is 248 Yards and 2 Feet.

137-6
16-3
922
137
34-4-6
8
2234

F.	I.	
137	6	
16	3	
830		
137		
34	4	6
9)2234	4	6
248	2	0

137.5
16.25
6875
2750
8250
1375
9)2234.375
248.2

137-6
16-3

Facit 248 Yards 2 Feet.

22

By

By Scale and Compaſſes.

For the firſt Example, extend the Compaſſes from 9 to 126.25, that Extent will reach from 15.75 to 220.9 Yards.

For the ſecond Example, extend the Compaſſes from 9 to 137.5, that Extent will reach from 16.25 to 248 Yards and about a Quarter.

In Joyners Work there is another thing to be obſerv'd, that is, in the meaſuring of Doors, Window-ſhutters, and all ſuch Work as is wrought on both Sides, they are paid for Work and Half-work; ſo that in meaſuring all ſuch Work, you muſt firſt find the Content, as before, and take half that Content, and add to it; ſo ſhall the Sum be the Content at Work and half.

Example. If the Window-ſhutters about a Room be 69 Feet 9 Inches broad, and 6 Feet 3 Inches high, How many Yards are contain'd therein at Work and half?

Multiply 69 Feet 9 Inches by 6 Feet 3 Inches, and the Product is 435 Feet 11 Inches 3 Parts; the Half whereof is 217 Feet 11 Inches 7 Parts; which added together, the Sum is 653 Feet 10 Inches 10 Parts; which divided by 9, the Quotient is 72 Yards 5 Feet, the Content at Work and half.

F.	I.		
69	9		69.75
6	3		6.25
418	6		34875
17	5	3	13950
			41850
435	11	3	
217	11	7	435.9375
			217.9687
9)653	10	10	
			653.9062
72	5		

Facit 72 Yards 5 Feet.

By Scale and Compaſſes.

Extend the Compaſſes from 9 to 69.75, that Extent will reach from 6.25 to 48.4 Yards; the Half whereof is 24.02; which added together, make 72.6 Yards, the Content at Work and half.

Note, That you muſt make Deductions for all Window-lights; but you muſt meaſure the Window-boards, Sopheta-boards, and Cheeks, by themſelves.

§ V. *Of* PAINTERS *Work.*

THE taking the Dimenſions of Painters Work is the ſame as that of Joyners, by girting over the Mouldings and ſwelling Panels, in taking the Height; and it is but Reaſon that they ſhould be paid for that on which their Time and Colour are both expended. The Dimenſions thus taken, the caſting up, and reducing Feet into Yards, is altogether the ſame as the Joyners Work; but the Painter never requires Work and half, but reckons his Work once, twice, or thrice colour'd over. Only take notice, that Window-lights, Window-bars, Caſements, and ſuch-like Things, they do at ſo much *per* Piece.

Example. If a Room be painted, whoſe Height (being girt over the Mouldings) is 16 Feet 6 Inches, and the Compaſs of the Room 97 Feet 9 Inches; How many Yards are in that Room?

Multiply 97 Feet 9 Inches by 16 Feet 6 Inches, and the Product is 1612 Feet 0 Inches 6 Parts; which being divided by 9, the Quotient is 179 Yards and 1 Foot.

F. I.
97 9
16 6

584
98
48 10 6

9)1612 10 6

179 1

97·75
16.5

48875
58650
9775

9)1612.875(179

97-9
16-6

582
97
48-10-6
8
4

1612·10·6

Facit 179 Yards 1 Foot.

By Scale and Compaſſes.

Extend the Compaſſes from 9 to 16.5, that Extent will reach from 97.75 to 179.2 Yards.

* * *

§ VI. *Of* GLASIERS *Work.*

GLaſiers do meaſure their Work by the Foot ſquare; ſo that the Length and Breadth of a Pane of Glaſs in Feet, being multiply'd into each other, produceth the Content.

Note, That Glaſiers do uſually take their Dimenſions to a Quarter of an Inch; and in multiplying Feet, Inches, and Parts, the Inch is divided into 12 Parts, as the Foot is, and each Part ſubdivided into 12, *&c.*

Example 1. If a Pane of Glaſs be 4 Feet 8 Inches and 3 Quarters long, and 1 Foot 4 Inches 1 Quarter broad; How many Feet of Glaſs are in that Pane?

The Decimal of $\left\{\begin{array}{l}8 \text{ Inches } \frac{3}{4}\\ 4 \text{ Inches } \frac{1}{4}\end{array}\right\}$ is $\left\{\begin{array}{l}.729\\ .354\end{array}\right.$

F.	I.	P.			
4	8	9			4.729
1	4	3			1.354
					18916
4	8	9			23645
1	6	11	0		14187
	1	2	2	3	4729
6	4	10	2	3	6.403066

Anſwer, 6 Feet 4 Inches.

By Scale and Compaſſes.

Extend the Compaſſes from 1 to 1.354, that Extent will reach from 4.729 to 6.4 Feet, the Content.

Example 2. If there be 8 Panes of Glaſs, each 4 Feet 7 Inches 3 Quarters long, and 1 Foot 5 Inches 1 Quarter broad; How many Feet of Glaſs are contained in the ſaid 8 Panes?

$$\text{The Decimal of}\left\{\begin{array}{l}7\text{ Inches }\frac{3}{4}\\5\text{ Inches }\frac{1}{4}\end{array}\right\}\text{ is }\left\{\begin{array}{l}.646\\.437\end{array}\right.$$

F.	I.	P.			
4	7	9			4.646
1	5	3			1.437
					32522
4	7	9			13938
1	11	2	9		18584
	1	1	11	3	4646
6	8	1	8	3	6.676302
				8	8
53	5	1.	6	0	53.410416

Facit 53 Feet 5 Inches.

By Scale and Compaſſes.

Extend the Compaſſes from 1 to 1.437, that Extent will reach from 4.646 to 6.676; then extend the Compaſſes from 1 to 8, that Extent will reach from 6.676 to 53.4, the Content.

Example 3. If there be 16 Panes of Glaſs, each 4 Feet 5 Inches and a half long, and 1 Foot 4 Inches 3 Quarters broad; How many Feet of Glaſs are contain'd therein?

F.	I.	P.			4.458
4	5	6			1.395
1	4	9			
					22290
4	5	6			40122
1	5	10	0		13374
	3	4	1	6	4458
6	2	8	1	6	6.218910
				4	4
24	10	8	6	0	24.875640
				4	4
99	6	10	0	0	99.502560

Facit 99 Feet 6 Inches.

Note, That inſtead of multiplying by 16, I have multiply'd by 4 twice, becauſe 4 times 4 is 16.

By Scale and Compaſſes.

Extend the Compaſſes from 1 to 1.395, that Extent will reach from 4.458 to 6.219; then extend the Compaſſes from 1 to 16, that Extent will reach from 6.219 to 99.5 Feet, the Content.

Note, That when Windows have Half-rounds at the Top, they meaſure them at the full Height, as if they were ſquare. Alſo round or oval Windows are meaſur'd ſur'd at the full Length and Breadth of their Diameters. Likewiſe Crotchet-windows in Stone-work are all meaſur'd by their full Squares. And there is Reaſon for ſo doing; for the Trouble in taking their Dimenſions to work by, the Waſte of Glaſs in Working, and the Time expended in ſetting up, is far more than the Glaſs can be valued at.

❁❁❁❁❁❁❁❁❁❁❁❁❁❁❁❁❁❁❁❁❁❁❁

§ V. *Of* MASONS *Work.*

MAſons do meaſure their Work ſometimes by the Foot ſolid, ſometimes by the Foot ſuperficial; and in ſome Places they meaſure their Walling by the Rood, that is, 21 Feet long, and 3 Feet high, which is 63 Square Feet. Examples of each are as follow.

Example 1. If a Wall be 97 Feet 5 Inches long, 18 Feet 3 Inches high, and 2 Feet 3 Inches thick; How many ſolid Feet are contain'd in that Wall?

12)18(1
6

F.	I.			
97	5			97.417
18	3			18.25
776				487085
97				194834
24	4	3		779336
6	0	0		97417
1	6	0		
				1777.86025
1777	10	3		2.25
2	3			
				888930125
3555	8	6		355572050
444	5	6	9	355572050
4000	2	0	9	4000.1855625

Multiply the Length, Height, and Thickneſs together, and the laſt Product is 4000 Feet 2 Inches, the ſolid Feet contain'd in the Wall.

By Scale and Compaſſes.

Extend the Compaſſes from 1 to 18.25, that Extent will reach from 97.417 to 1777.86; then extend from 1 to 1777 86, that Extent will reach from 2.25 to 4000.18, the ſolid Content.

Example 2. If a Wall be 107 Feet 9 Inches long, and 20 Feet 6 Inches high; How many Feet ſuperficial are contain'd therein?

F.	I.		
107	9		107.75
20	6		20.5
2155	0		53875
53	10	6	215500
2208	10	6	2208.875

Facit 2208 Feet 10 Inches.

By Scale and Compaſſes.

Extend the Compaſſes from 1 to 107.75, that Extent will reach from 20.6 to 2208.875, the ſuperficial Feet.

107–9
20–6

2140
53–10–6
15
0 [illegible] 10–6

Example 3. If a Wall be 112 Feet 3 Inches long, and 16 Feet 6 Inches high; How many Roods are contain'd therein?

F. I.
112 3
16 6
676 0
112
56 1 6
1852 1 6

112.25
16.5
56125
67350
11225
63)1852.125(29
592
25

Facit 29 Roods 25 Feet.

By Scale and Compaſſes.

Extend the Compaſſes from 63 to 16.5, that Extent will reach from 112.25 to 29.4 Roods, the Content.

112-3
16-6
672 0
112
56-1-6
.4
1852:1'6

CHAP. IV.

The Meaſuring of BOARD *and* TIMBER.

§ I. *Of* BOARD MEASURE.

TO meaſure a Board, is no other but to meaſure a long Square.

Example 1. If a Board, be 16 Inches broad, and 13 Feet long; How many Feet are contain'd therein?

Multiply 16 by 13, and the Product is 208; which divided by 12, gives 17 Feet, and 4 remains, which is a third Part of a Foot.

Or thus: Multiply 156 (the Length in Inches) by 16, and the Product is 2496; which divided by 144, the Quotient is 17 Feet, and 48 remains, which is a third Part of 144, the ſame as before.

12 : 13 : : 16
13

48
16

12)208

$17\frac{4}{12}$

1. 4
1. 4
1 4
5 4
1. 9. 4
12
2 1. 4 0
1. 4
32. 8 0

Or, 144 : 156 : : 16
16

936
156

144)2496($17\frac{48}{144}$
1056

48

By Scale and Compaſſes.

Extend the Compaſſes from 12 to 13, that Extent will reach from 16 to $17\frac{1}{3}$ Feet, the Content.

Or, extend from 144 to 156 (the Length in Inches) that Extent will reach from 16 to $17\frac{1}{3}$ Feet, the Content.

Example 2. If a Board be 19 Inches broad; How many Inches in Length will make a Foot?

Divide 144 by 19, and the Quotient is 7.58 very near; and ſo many Inches in Length, if a Board be 19 Inches broad, will make a Foot.

Inch. Inch. Inch. Inch.
19 : 144 : : 1 : 7.58 *fere.*

Extend the Compaſſes from 19 to 144, that Extent will reach from 1 to 7.58; that is, 7 Inches and ſomething more than a half. So, if a Board be 19 Inches broad, if you take 7 Inches, and a little more than a half with your Compaſſes from a Scale of Inches, and run that Extent along the Board, from End to End, you may find how many Feet that Board contains; or you may cut off from that Board any Number of Feet deſir'd.

For this Purpoſe there is a Line upon moſt ordinary Joint rules, with a little Table plac'd upon the End of all ſuch Numbers as exceed the Length of the Rule, as in this little Table annex'd.

0	0	0	0	5	0	8½	6
12	6	4	3	2	2	1	1
1	2	3	4	5	6	7	8

Here you ſee, if the Breadth be one Inch, the Length muſt be 12 Feet; if two Inches, the Length is 6 Feet; if 5 Inches broad, the Length is 2 Feet 5 Inches, *&c.*

The reſt of the Lengths are expreſs'd in the Line, thus: If the Breadth be 9 Inches, you will find it againſt 16 Inches, counted from the other End of the Rule; if the Breadth be 11 Inches, then a little above 13 Inches will be the Length of a Foot, *&c.*

§ II. *Of* SQUAR'D TIMBER.

BY Squar'd Timber are here meant ſuch as have equal Baſes, and the Sides ſtrait and parallel. The Rules for meaſuring all ſuch Solids are ſhew'd in § II. of Chap. 2. to which I refer you.

Example 1. If a Piece of Timber be 1 Foot 3 Inches (or 15 Inches) ſquare, and 18 Feet long; How many ſolid Feet are contain'd therein?

15	F.	I.	
15	1	3	
	1	3	
75			
15	1	3	
		3	9
225			
18	1	6	9
			6
1800			
225	9	4	6
			3
144)4050(28.125			
	28	1	6
1170			
180			
360			
720.			

Anſwer, 28 Feet and half a Quarter.

Here, inſtead of multiplying by 18, where wrought by Feet and Inches, I multiply'd by 6, and then by 3, becauſe 3 times 6 is 18.

Example 2. If a Piece of ſquar'd Timber be 2 Fee 9 Inches deep, and 1 Foot 7 Inches broad, and 1 Feet 9 Inches long; How many Feet of Timber ar in that Piece?

Multiply the Depth, Breadth, and Length togethe and the Product will be the Content.

33
19

297
33

627
16.75

3135
4389
3762
627

144)10502.25(72.93

422
1342
465

33

F. I.
2 9
1 7

2 9
1 7 3

4 4 3
16 8 0

69 9 0
3 3 2 3

72 11 2 3

4 - 4 - 3
16 - 8 - 0
64
1 - 4 - 5
1 - 4 - 5
5
6

Anſwer 72 Feet 11 Inches; or, 72 Feet 93 Parts.

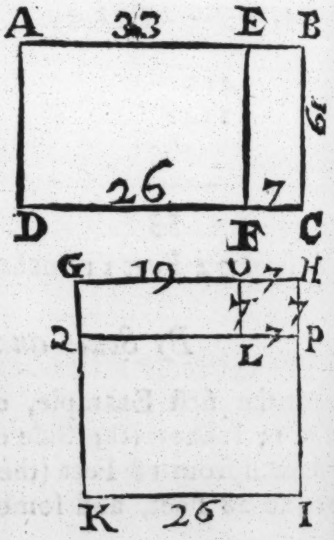
By Scale and Compaſſes.

For the firſt Example, extend the Compaſſes from 2 to 15 Inches (the Side of the Square) that Extent ill reach from 18 Feet (the Length being twice turn'd ver) to 28 Feet, and ſomething more.

For the ſecond Example, find a mean Proportional etween 19 Inches and 33 Inches, by dividing the pace between them into two equal Parts; and the ompaſs Point will reſt upon 25, which is a mean roportional between 19 and 33.

Then Extend the Compaſſes from 12 to 25 (the roportional found) that Extent will reach (being twice rn'd over) from 16.75 Feet, the Length, to 72.93 eet, the Content.

A common Error is committed, for want of Art, in meaſuring theſe laſt Sorts of Solids, by adding the Depth and Breadth together, and taking half for the Side of a mean Square. This Error, though it be but ſmall, when the Depth and Breadth are pretty near equal; yet if the Difference be great, the Error is very conſiderable; for the Piece of Timber thus meaſur'd, will be more than the Truth, by a Piece whoſe Length is equal to the Length of the Piece of Timber to be meaſur'd, and the Square equal to half the Difference of the Breadth and Depth, as I ſhall here demonſtrate.



I ſay, the Square GHIK is greater than the Parallelogram A B C D, by the little Square O H P L; for the Parallelogram Q P I K is equal to the Parallelogram AEFD; and the Parallelogram G O L Q is equal to the Parallelogram E B C F. Therefore the Square is greater than the Parallelogram by the little Square OHPL; which was to be prov'd.

Otherwiſe, you may prove it by Numbers, thus; the Sum of 33 and 19 is 52; the half thereof is 26; the Square of 26 is 676: And the Product of the Depth and Breadth is 627; the Difference of theſe two is 49, equal to the Square of half the Difference; for the Difference between 33 and 19 is 14, the half thereof is 7, whoſe Square is 49; which was to be prov'd.

Now, if this 49 be multiply'd by the Length of the Piece, and that Product divided by 144, to bring it to Feet, and thoſe Feet added to the true Content, the

Sum will be equal to the Content found by the falſe Way mentioned.

See the Work of both.

33 Depth.
19 Breadth.
52 Sum.
26 half
26
156
52
676
16.75
3380
4732
4056
676
144)11323.00(78.63
1243
910
460
28

16.75 the Length.
49 the Squ. of ½ Diff.
15075
6700
144)820.75(5.69
1007
1435
139

Feet.
To 72.93 the true Content.
Add 5.69 the Part ſuperfluous.

Rem. 78.62 equal to the Content by the falſe Way.

F.	I.			
0	7			
0	7			
0	4	1		
16	9			
5	5	4		
	3	0	9	
5	8	4	9	Part ſuperfluous.
72	11	2	3	true Content add.
78	7	7	0	equal to the Content by the falſe Way.

By Feet and Inches.

	F.	I.	
	2	2	
	2	2	
	4	4	
		4	4
	4	8	4
	16	9	
	75	1	4
	3	6	3
Falſe C.	78	7	7

To find how much in Length makes a Foot of any ſquar'd Timber.

Always divide 1728 (the ſolid Inches in a Foot) by the Area of the Baſe; the Quotient is the Length of a Foot.

This Rule is general for all Timber, which is of equal Thickneſs from d to End, whether it be ſquare, triangular, multangular, or round.

Example 1. If a Piece of Timber be 18 Inches ſquare, How much in Length will make a Foot ſolid?

18
18

144
18

324(1728($5\frac{1}{3}$
1620

108 Anſwer, 5 Inches and $\frac{1}{3}$.

By Scale and Compaſſes.

Extend the Compaſſes from 1 to 18, that Extent will reach from 18 to 324, the Square or Area of the Baſe; then extend from 324 to 1728, that Extent will reach down from 1 to 5 Inches, and $\frac{1}{3}$ of an Inch

Or thus: Extend the Compaſſes from 18 to 41 569, that Extent, turn'd twice over from 1, will at laſt fall upon 5 $\frac{1}{3}$, as before.

Note, That 41.569 is the ſquare Root of 1728.

Example 2. If a Piece of Timber be 22 Inches deep, and 15 Inches broad, How much in Length will make a Foot?

22
15
———
110
22
———
330)1728(5.23
———
780
1200
———
210

Anſwer, 5 Inches and .23 Parts.

By Scale and Compaſſes.

Extend the Compaſſes from 1 to 15, that Extent will reach from 22 to 330; then extend from 330 to 1728, that Extent will reach from 1 to 5.23 Inches, the Length of a Foot.

There is a Line for this Purpoſe upon moſt ordinary Rules, with a little Table at the End, of all ſuch Numbers as exceed the Length of the Rule, ſuch as this annex'd.

0	0	0	0	9	0	11	3	9	Inches.
144	36	16	9	5	4	2	2	1	Feet.
1	2	3	4	5	6	7	8	9	Side of the Sq.

Here you ſee, if the Side of the Square be 1, the Length muſt be 144 Feet; if two Inches be the Side of the Square, it muſt be 36 Feet in Length, to make a ſolid Foot, *&c.*

If the Side of the Square be not in the little Table, you will find it upon the Line; thus, if the Side of the Square be 16 Inches, you will find it againſt 6 Inches and 7 Tenths, counted from the other End of the Rule.

Then if you take the Length of a Foot from the Line of Inches with your Compaſſes, and run the Compaſſes along the Piece, from End to End, you will find how many Feet are contain'd in that Piece; or you may cut off any Number of ſolid Feet that ſhall be deſir'd; but if the Sides of the Piece be unequal, find a mean proportionable Number, as is before taught, by dividing the Diſtance upon the Line of Numbers into two equal Parts: Thus, if the Breadth be 25 Inches, and the Depth 9 Inches, divide the Space upon the Line of Numbers, into two equal Parts, and you will find the middle Point at 15; ſo is 15 Inches the geometrical mean Proportional ſought; then if you look for 15 upon the Line above-mention'd, you will find 7 Inches, and a little above half, to be the Length of a Foot.

§ II

§ III. *Of unequal Squar'd Timber.*

BY unequal Squar'd Timber, I mean all ſuch as have unequal Baſes; that is, ſuch as is thicker at one End than at the other; and ſuch are moſt Timber-trees when they are hewn, and brought to their Squares.

The uſual Way to meaſure ſuch Timber, is to take a Square about the Middle of the Piece, which they take to be a mean Square: This Way, when the Piece is pretty near as thick at one End as at the other, is ſomething near the Truth; but when there is a great Diſproportion between the Ends of the Piece, the Error is conſiderable. All ſuch Solids being the Fruſtums of Pyramids, the true Way of meaſuring them muſt be by Sect. VII. Chap. 2. I ſhall give an Example or two, which I will work both by the true and falſe Ways, whereby you will ſee the Difference.

Example 1. If a Piece of Timber be 25 Inches ſquare at the greater End, and 9 Inches ſquare at the leſſer End, and 20 Feet long; How many Feet of Timber are in that Tree?

```
      25
       9
Sum   34
Half  17 the Side of the Square in the Middle.
      17
     119
      17
     289
      20
 144)5780(40.13
      200
       560
       128
```

Anſwer, 40.13 Feet, by the falſe Way.]

By Rule II. Sect. VII. Chap. 2.

25 25
9 9
225 16 Difference of the Sides.
16
96
16
3)256 the Square.
85.333
225
310.333
20
144)6206.660(43.101
446
146
260
116

Anſwer, 43.101 Feet, by the true Way; ſo tha
there is near 3 Feet Difference.

By Scale and Compaſſes.

Extend from 1 to 9, that Extent will reach fro
25 (the ſame Way) to 225, the Rectangle of the Side
of the two Baſes; then the Difference between th
ſaid Sides is 16; extend from 3 to 16, that Exte
will reach from 16 to 85.333, a third Part of th
Square; which added to 225, the Sum is 310.333,
mean Area: then extend from 144 to 310.333, th

Exte

Extent will reach from 20 (the Length) to 43.1 Foot the Content the true Way.

Extend the Compaſſes from 12 to 17, (the Side of the middle Square) that Extent will reach from 20, (the Length being twice turn'd over) to 40.1 Foot, the Content by the falſe Way.

Example 2. If a Piece of Timber be 32 Inches broad, and 20 Inches deep, at the greater End, and 10 Inches broad, and 6 deep, at the leſſer End, and 18 Foot long; How many Feet of Timber are in that Piece?

Rule I. Sect. VII. Chap. 2.

```
 32             6
  20           10
 ----          --
 640           60
   60
 ------
 .  .  .
 38400          (195.959 mean Proportional.
 1               640     the greater Baſe.
 ------           60     the leſſer Baſe.
29)284          --------
 385)2300        895.959 the Sum.
 3909)37500            6 1/3 Height.
 39185,231900   --------
 391909)3597500 5375.754

        144)5375.754(37.33
            --------
            1055
              477
               455
            ------
                23
```

And

Add $\left\{\begin{array}{r}32\\10\end{array}\right.$ $\left.\begin{array}{r}20\\6\end{array}\right\}$ Add

Sum 42 26 Sum

Half 21 13 Half.

```
    21
    13
    ——
    63
   21
   ———
   273 Area in the Middle.
    18 Length.
  ————
  2184
  273
  ————
144)4914(34.12
  ————
   594
    180
     360
   —————
      72
```

Feet.

Anſwer { Content the true Way 37.33
Content the falſe Way 34.12 }

By Scale and Compaſſes.

Extend the Compaſſes from 1 to 20, that Extent will reach from 32 to 640, the Area of the greater Baſe.

Then extend from 1 to 10, that Extent will reach from 6 to 60, the Area of the leſſer Baſe: Then extend from 1 to 60, that Extent will reach from 640 to 38400, the Product of the two Areas: Find the Square Root thereof, by dividing the Space between 1 and 38400 into two equal Parts, ſo you will find the middle Point at 195.959, the Root ſought; which

i

is a mean Proportional between the greater and leſſer Areas; then add the mean Proportional and two Areas together, and the Sum is 895.959; which multiply'd by 6 (a third Part of the Length), by extending from 1 to 6, that Extent will reach from 895.959 to 5375.75. Then extend from 144 to 5375.75, and that Extent will reach from 1 to 37.33 Feet, the true Content.

For the falſe Way, half the Sum of the Breadths is 21, which is the Breadth in the Middle; and half the Sum of the Depths is 13: Extend from 1 to 13, that Extent will reach from 21 to 273, the Area of the middle Baſe: Then extend from 144 to 273, that Extent will reach from 18, the Length, to 34.12, the Content the falſe Way.

* * *

§ IV. *Of Round Timber, whoſe Baſes are equal.*

THE uſual Way to meaſure Round Timber-trees is to girt them about the Middle with a String, and take the fourth Part of that Girth for the Side of a Square, by which they meaſure the Piece of Timber as if it was ſquare.

But that this is an Error, I ſhall make appear as follows. If the Circumference of a Circle be 1, the Area will be .07958; then the fourth Part of 1 is .25, which ſquar'd makes .0625; this they take for a mean Area, inſtead of .07958: Therefore the true Content always bears ſuch Proportion to the Content found by the aforeſaid cuſtomary falſe Way, as .07958 to .0625; which is nearly as 23 to 18; ſo that in meaſuring by that cuſtomary falſe Way, there is above the one fifth Part loſt, of what the true Content ought to be.

This Error, tho' it has been ſo often confuted, yet it is grown ſo cuſtomary in all Places, that there is little Hope of my prevailing with Men that are ſo wedded to it, to embrace the Truth: I ſhall therefore in the following Examples, ſhew how to work both the true Way, and alſo the falſe or cuſtomary Way.

Example 1. If a Piece of Timber be 96 Inches in Circumference or Girth, and 18 Feet long; How many Feet of Timber are contain'd therein?

A fourth Part of 96 is 24
24
—
96
48
—
576 Area Baſe,
18
—
4608
576
—
144)10368(72
1008
—
288
288
—
...

Or thus,

F.	I.
2	0
2	0
4	0
18	
72	0

2,4
2,4
4.9
9.4
5,5.4
6
32,8 0
3
98„0„0

Content the falſe Way, 72 Feet.

Then the true Way.

```
          96
          96
        ----
         576
        864
        ----
        9216
      .07958
      ------
       73728
      46080
     82944
    64512
    ---------
    733.40928 The Area by Prob. 5. Sect. IX. Ch. 2.
           18
    ---------
    586727424
    73340928
    ---------
144)13201.36704(91.67
    ------
     241
      973                The true Content 91.67 Feet.
      1096
      ----
        88
```

By Scale and Compaſſes.

Extend from 12 to 24 (the fourth Part of the Girth), that Extent turn'd twice over from 18 Feet (the Length), will at laſt fall upon 72 Feet, the Content the cuſtomary Way.

Extend from 42.54 to 96 (the Girth), that Extent will reach from 18 Feet, turn'd twice over, to 91.67 Feet, the true Content.

Example 2. If a Piece of Timber be 86 Inches Girth, and 20 Feet long; How many Feet are contain'd therein?

The fourth Part of 86 is 21.5

F.	I.	P.	
1	9	6	
1	9	6	
1	9	6	
1	4	1	6
	0	10	9
3	2	6	3
			20
64	2	5	0

21.5
21 5
1075
215
430
462.25
20
144)9245.00(64.2
605
290
20

The Content the falſe Way, 64.2 Feet.

3-2-6-3
5
16-0-7-3
4
64-2-5-0

By the true Way.

```
        86
        86
      ----
       516
      688
      ----
      7396
    .07958
    ------
     59168
    36980
   66564
  51772
  --------
  588.57368
          20
  ----------
144)11771.47360(81.74
    ------
    251
    1074
      667
    -----
      91
```

The true Content, 81.74 Feet.

By Scale and Compaſſes.

Extend from 12 to 21.5, that Extent turn'd twice over from 20, will reach at laſt to 64.2 Feet, the Content the falſe Way.

Extend from 42.54 to 86, that Extent, turn'd twice over from 20, will at laſt fall upon 81.74 Feet, the true Content.

Theſe Cylindrical Proportions may be very eaſily wrought upon the Line of Numbers.

Problem 1. Having the Diameter of a Cylinder in Inches, to find the Length of a Foot.

Suppoſe the Diameter 22.6 Inches.

As 22.6 is to 46.9, ſo is 1 to a fourth Number; and that to the Length of a Foot in Inches 4.3.

Extend the Compaſſes from 22.6 to 46.9, that Extent will reach from 1 to a fourth Number; then turn them over again, and that will reach to 4.3 Inches.

Problem 2. Having the Diameter in Foot-meaſure, to find the Length of a Foot in Foot-meaſure.

Suppoſe the Diameter 1.88 Feet.

Then, as 1.88 is to 1.128, ſo is 1 to a fourth Number; and ſo is that to the Length of a Foot in Foot-meaſure, .358.

Extend the Compaſſes from 1.88 to 1.128, that Extent, turn'd twice from 1, will reach to .358 Parts of a Foot.

Problem 3. Having the Circumference in Inches, to find the Length of a Foot in Inches.

Suppoſe the Circumference 71 Inches.

Then, as 71 is to 247.36, ſo is 1 to a fourth Number; and ſo is that to the Length of a Foot in Inches, 4.3.

Extend the Compaſſes from 71 to 147.36, that Extent, turn'd twice from 1, will reach to 4.3 Inches, the Length of a Foot.

Problem 4. Having the Circumference in Foot-meaſure, to find the Length of a Foot in Foot-meaſure.

Suppoſe the Circumference 5.92 Feet.

Then, as 5.92 is to 3.345, ſo is 1 to a fourth Number; and ſo is that to the Length of a Foot in Foot-meaſure, .358.

Exten

Extend the Compaſſes from 5.92 to 3.545, tha Extent, turn'd twice over from 1, will fall upon .358 Parts of a Foot.

Problem 5. Having the Diameter in Inches, and the Length in Inches, to find the Content in Inches.

Suppoſe the Diameter 22.6 Inches, and the Length 156 Inches, or 13 Feet.

Then, as 1.128 is to 22.6, ſo is 156 to a fourth Number; and ſo is that to the Content in Inches, 62674.

Extend the Compaſſes from 1.128 to 22.6, that Extent, turn'd twice from 156, will fall upon 62674 Inches, the Content.

Note, That 1.128 is the Diameter when the Side of the Square is equal to 1.

Problem 6. Having the Diameter in Foot meaſure, and Length in Feet, to find the Content in Feet.

Suppoſe the Diameter 1.88 Feet, and the Length 13 Feet.

Then, as 1.128 is to 1.88, ſo is 13 to a fourth Number; and ſo is that to the Content in Feet 36.27.

Extend from 1.128 to 1.88, that Extent, turn'd twice from 13, will fall upon 36.27.

Problem 7. Having the Diameter in Inches, and Length in Inches, to find the Content in Feet.

Suppoſe the Diameter 22.6 Inches, and the Length 156 Inches.

Then, as 46.9 is to 22.6, ſo is 156 to a fourth Number; and ſo is that to the Content in Feet, 36.27.

Extend from 46.9 to 22.6, that Extent, turn'd twice from 156, will fall upon 36.27 Feet, the Content.

Note, That 46.9 is the Diameter of a Circle, whoſe Area is 1728.

Problem 8. Having the Diameter in Inches, and Length in Feet, to find the Content in Feet.

Suppoſe the Diameter 22.6 Inches, and the Length 13 Feet.

Then, as 13.54 is to 22.6, ſo is 13 to a fourth Number; and ſo is that to the Content in Feet, 36.27.

Extend from 13.54 to 22.6, that Extent, turn'd twice from 13, will fall upon 36.27.

Note, That 13.54 is the Diameter of a Circle, when the Area is 144.

Problem 9. Having the Circumference in Inches, and Length in Inches, to find the Content in Inches.

Suppoſe the Circumference 71, and the Length 156 Inches.

Then, as 3.545 is to 71, ſo is 156 to a fourth Number; and ſo is that to 62674, the Content in Inches.

Extend the Compaſſes from 3.545 to 71, that Extent turn'd twice from 156, will fall upon 62674, the Content.

Note, That 3.545 is the Circumference, when the Side of the Square is equal to 1.

Problem 10. Having the Circumference in Feet, and Length in Feet, to find the Content in Feet.

Suppoſe the Circumference 5.92 Feet, and Length 13 Feet.

Then, as 3.545 is to 5.92, ſo is 13 to a fourth Number; and ſo is that to 36.27.

Extend from 3.545 to 5.92, that Extent turn'd twice from 13, will fall upon 36.27 Feet, the Content.

Problem 11. Having the Circumference in Inches, and Length in Inches, to find the Content in Feet.

Suppoſe the Circumference 71 Inches, and Length 156 Inches.

Then, as 147.36 is to 71, ſo is 156, to a fourth Number; and ſo is that to the Content in Feet 36.27.

Extend the Compaſſes from 147.36 to 71, that Extent, turn'd twice from 156, will fall upon 36.27 Feet, the Content.

Note, That 147.36 is the Circumference of a Circle, whoſe Area is 1728.

Problem 12. Having the Circumference in Inches, and Length in Feet, to find the Content in Feet.

Suppoſe the Circumference 71 Inches, and Length 13 Feet.

Then, as 42 54 is to 71, ſo is 13 to a fourth Number; and ſo is that to the Content in Feet 36.27.

Extend the Compaſſes from 42 54 to 71, that Extent, turn'd twice from 13, will reach to 36.27 Feet, the Content.

Note, That 42.54 is the Circumference of a Circle, whoſe Area is 144.

§ V. *Of Round Timber, whoſe Baſes are unequal.*

THE uſual Way to meaſure Round Timber (as I ſaid before) is to take a fourth Part of the Girth in the Middle of the Piece, for the Side of a mean Square. But this Way I have prov'd to be erroneous in Timber that is all the Way of an equal Thickneſs; and it muſt be much more ſo in Timber that is tapering; and the more tapering it is, the greater is the Error: for to the Error in the laſt Section, there is added the Error in the third Section; therefore, to meaſure all ſuch Timber according to Art and Truth, ſuch a Piece ought to be conſider'd as a Fruſtrum of a Cone, and ſhould be meaſur'd by the Rules given in Section VIII. Chapter 2. by which Rules the following Examples are wrought.

Example 1. If a Piece of Timber be 9 Inches Diameter at the leſſer End, and 36 Inches at the other End, and 24 Feet long; How many Feet of Timber are therein?

```
        36                 36 } Subtract.
         9                  9 }
       ---                 --
Rect. 324                  27 Difference.
                           27
                          ---
                          189
                          54
                         ----
                        3)729 The Square.
                         ----
                          243 One Third.
                          324 Rectangle add.
                         ----
                          567
```

.7854

.7854
567

54978
47124
39270

A mean Area 445.3218
24

17812872
8906436

144)10687.7232(74.22
607
317
292

4

Anſwer, 74.22 Feet.

Or thus, by Feet and Inches.

F.	I.		F.	I.		
3	0		2	3		Difference.
0	9		2	3		
2	3	Rect.	4	6		
			0	6	9	
			5	0	9	The Square.
			1	8	3	One Third.
			2	3	0	Rectangle added.
			3	11	3	A mean Square.

Then

F. I. P.
Then, as 14 is to 11, ſo is 3 11 3 to the Area.
11

7)43 3 9

2) 6 2 3

3 1 1 6
6

18 6 9 0
4

74 3 0 0

Here, inſtead of dividing by 14, I divide by 7 and by 2, becauſe twice 7 is 14.

And inſtead of multiplying by 24 Feet, the Length, I multiply by 6 and by 4, becauſe 6 times 4 is 24.

By Scale and Compaſſes this is too troubleſome.

Example 2. If a Piece of Timber be 136 Inches Circumference at one End, and 32 Inches Circumference at the other End, and 21 Feet long; How many Feet of Timber are contain'd in that Piece?

136
32
272
408
4352

136
32
104 Difference.
104
416
1040
3)10816 The Square.
3605.333 One Third.
4352 Rectangle add.
7957.333 A meanCircu.ſquar'd.
.07958
63658664
39786665
71 15997
55701331
633.24456014 The mean Area.
21
63324456014
126648912028
13298.13576294

144)13298.13(92.34
338
501
693
117

Anſwer, 92.34 Feet.

By Feet and Inches, thus;

F.	I.	
11	4	
2	8	
22	8	
7	6	8
30	2	8

	F.	I.			
	8	8			Difference.
	8	8			
	69	4			
	5	9	4		
3)	75	1	4		The Square.
	25	0	5	4	
	30	2	8	0	
	55	3	1	4	The Sq. of the Circum.
	F.	I.	P.	S.	
88 : 7 ::	55	3	1	4	: The mean Area.
				7	
11)	386	9	9	4	
8)	35	1	11	9	
	4	4	8	11	The mean Area.
				7	
	30	9	2	5	
				3	
Facit	92	3	7	3	

§ VI

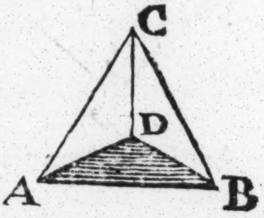
§ VI. *Of the Five Regular Bodies.*

THESE Bodies may all be meaſur'd by the 4th Section of Chap. 2. except it be the Cube, or Hexaedron, which is already meaſur'd in Section I. of that Chapter.

Of the TETRAEDRON.

A Tetraedron is a Solid, contain'd under four equal and equilateral Triangles.

Let ABCD be a Tetraedron, whoſe Side is 12 Inches, the perpendicular Height 9.798 Inches.



By Sect. V. Chap. 1. the Area of the Triangle will be found 62.352 ; a third Part of it is 20.784, which multiply'd by the perpendicular Height, the Product is 203.641632 ſolid Inches, the Content.

10.392 The Perpendicular of the Triangle.
6 Half the Side.

62.352 Area of the Triangle.

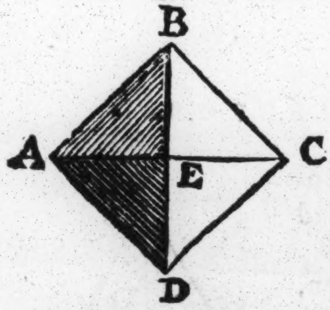
20 784 One third Part.
9.798 The perpendicular Height.

166272
187056
145488
187056

203.641632

The ſuperficial Content is four times the Area of the Triangle, *viz.* 249.408 Inches, becauſe there are 4 Triangles.

2. *Of the* OCTAEDRON.



The Octaedron is a Body contain'd under eight equal and equilateral Triangles.

Let ABCDE be an Octaedron, whoſe Side is 12 Inches; the Content ſolid and ſuperficial is requir'd.

An Octaedron is compos'd of two quadrangular Pyramids join'd together by their Baſes; therefore, if the Area of the Baſe be multiplied into a third Part of the Length of both Pyramids, the Product will be the ſolid Content.

5.6568 A third Part of the Length.
144 Area of the Square Baſe.

226272
226272
56568

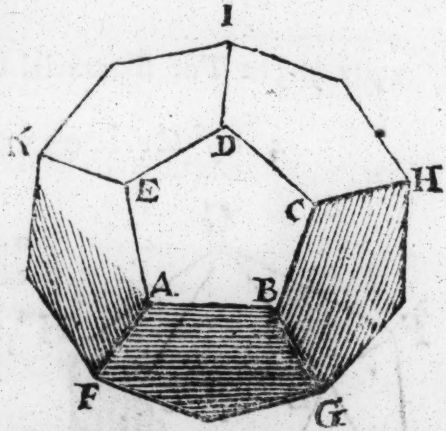
814.5792 The Solidity.

The ſuperficial Content will be juſt double to that of the Tetraedron, *viz.* 498.816, becauſe the Side of this is ſuppos'd to be equal to the Side of that, and becauſe the Octaedron is contain'd under eight Triangles, and the Tetraedron but under four.

3. *Of the* DODECAEDRON.

The Dodecaedron is a ſolid Body, contained under twelve pentangular Plains.

Let ABCDEFGHIK be a Dodecaedron, each Side thereof being 12 Inches; the Content ſolid and ſuperficial is required.



The Solidity of the Dodecaedron is compos'd of 12 pentangled Pyramids, whoſe Vertexes all meet in the Centre. Therefore, if we find the Solidity of one of thoſe Pyramids, and multiply that by 12, that Product will be the Solidity of the Dodecaedron.

The Altitude of one of the pentangled Pyramids will be found to be 13.36219.

The Perpendicular of the Pentagon will be

8.258292
30 Half Sum of the Sides.

247.748760 Area of the Pentagon.
60454.4 A third Part of 13.36219 inverted.

99099504
9909950
1238744
99099
1486

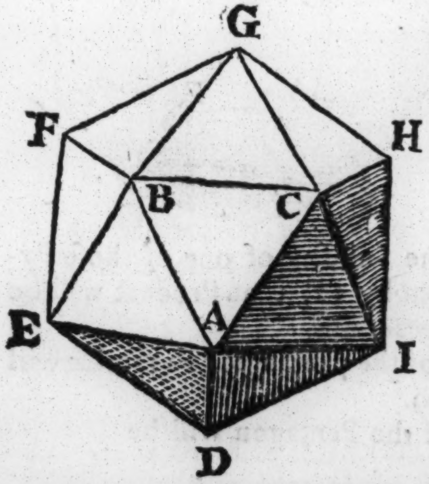
1103.48783 Content of one Pyramid.
12

13241.85396 The Solidity of the Dodecaedron.

If the Area of the Pentagon be multiply'd by 12, the Product will be the ſuperficial Content.

```
 247.74876
        12
 ---------
2972.98512 The ſuperficial Content.
```

4 *Of the* ICOSAEDRON.



The Icoſaedron is a ſolid Body, contain'd under twenty equal and equilateral Triangles.

Let ABCDEFGHI be an Icoſaedron, each Side thereof being 12 Inches; the Content ſolid and ſuperficial is required.

The Icoſaedron is compos'd of twenty triangular Pyramids, with their Vertexes all joining in the Centre.

Therefore, if the ſolid Content of one Pyramid be multiply'd by 20, the Product is the whole ſolid Content of the Icoſaedron.

10.39224 The Perpendicular of the Triangle.
6 Half the Side.

62.35344
20

1247.06880

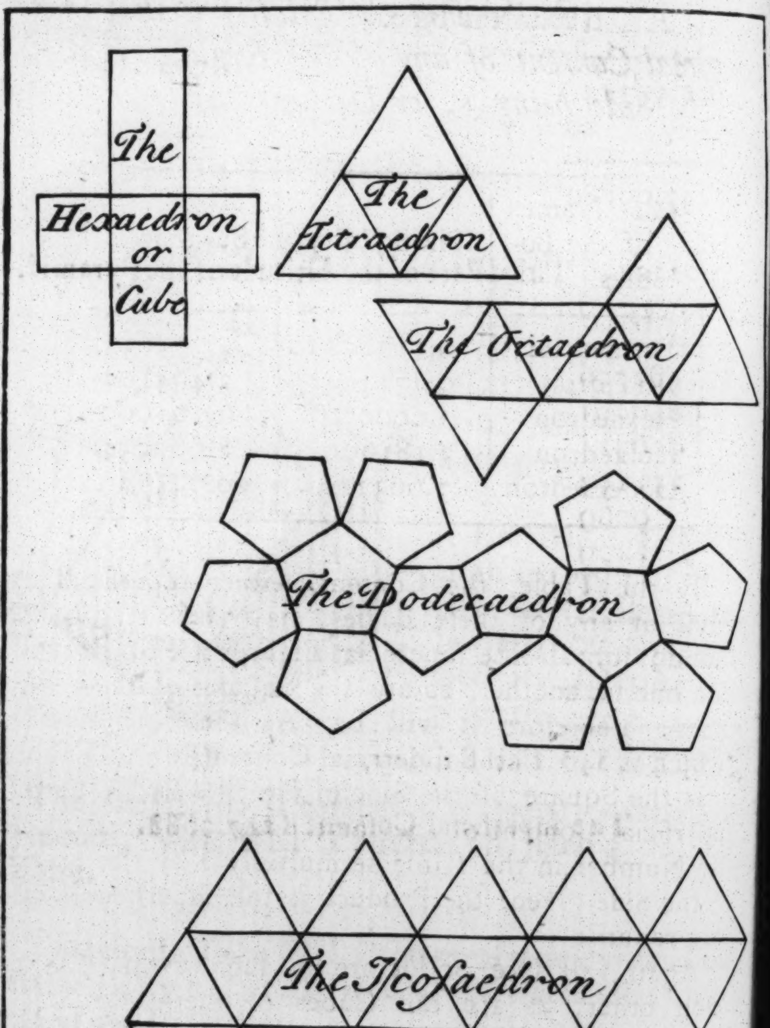
3.0230456 Third Part of the Altitude of the Pyramid.
44353.26

181382736
6046091
906914
151152
9069
1209
121

188.497292
20

3769.945840 The Solidity,

The ſuperficial Content 1247.0688.



By theſe Figures you may cut theſe Bodies in fin Paſteboard, cutting all the Lines half through, and ſ turn them up and glue them.

A TABLE *ſhewing the Solidity and Superficial Content of any of the Regular Bodies, the Side being* 1, *or Unity.*

The Names of the Bodies.	The Solidity.	Superficies.
Tetraedron	0.1178511	1.732051
Octaedron	0.4714045	3.464102
Hexaedron	1,0000000	6.000000
Icoſaedron	2.181695	8.660254
Dodecaedron	7.663119	20.645729

By this Table, the Content, either ſuperficial or ſolid, of any of theſe Bodies, may very readily be found; for all like ſuperficial Figures are in Proportion one to another, as are the Squares of their like Sides: Therefore it will be, As the Square of 1 (which is 1) is to the ſuperficial Content in the Table; ſo is the Square of the Side of the like Body, to the ſuperficial Content of the ſame Body. Therefore, if the Number in the Table be multiply'd by the Square of the Side given, the Product is the ſuperficial Content requir'd.

Again, all like Solids are in ſuch Proportion to each other, as are the Cubes of their like Sides. Therefore it will be as 1 (which is the Cube of 1) is to the ſolid Content in the Table, ſo is the Cube of the Side given, to the ſolid Content requir'd. Therefore, if the Number in the Table be multiply'd by the Cube of the given Side, the Product will be the ſolid Content of the ſame Body.

Example 1. If the Side of a Dodecaedron be 12 Inches (as before), what is the Content ſolid and ſuperficial?

7.663119 The tabular Number.
1728 The Cube of the Side.

61304952
15326238
53641833
7663119

13241.869632 The ſolid Content, nearly the ſame as (before.

20.645729 The tabular Number.
144 The Square of the Side.

82582916
82582916
20645729

2972.984976 The ſuperficial Content.

By Scale and Compaſſes.

Extend from 1 to 12 (the Side), that Extent being turn'd three times over from 7.63119, will at laſt fall upon 13241.86, *&c.* the ſolid Content.

And if you apply the ſame Extent twice from 20.645729, it will at laſt fall upon 2972.98, *&c.* the ſuperficial Content.

Example 2. If the Side of an Octaedron be 20 Inches, what is the Content ſolid and ſuperficial?

.4714045 The tabular Number
8000 The Cube of the Side.

3771.2360000 The ſolid Content.

3.464102 The tabular Number.
400 The Square of the Side.

1385.640800 The ſuperficial Content.

By Scale and Compaſſes.

Extend from 1 to 20, that Extent, turn'd three times over from .4714045, will at laſt fall upon 3771.236, the ſolid Content. The ſame Extent turn'd twice over from 3.464, *&c.* will at laſt fall upon 1385.64, the ſuperficial Content.

* * *

§ VII. *How to meaſure any irregular Solid.*

IF you have any Piece of Wood or Stone that is craggy and uneven, and you deſire to find the Solidity, put the Solid into any regular Veſſel, as a Tub, a Ciſtern, or the like, and pour in as much Wa- as will juſt cover it; then take out the Solid, and meaſure how much the Fall of the Water is, and ſo find the Solidity of that Part of the Veſſel.

Example. Suppoſe a Piece of Wood or Stone to be meaſur'd, and ſuppoſe a Tub 32 Inches Diameter, into which let the Stone or Wood be put, and cover'd with Water; then, when the Solid is taken out, ſuppoſe the Fall of the Water 14 Inches; Square 32, and multiply the Square by .7854, the Product will be 804.2496, the Area of the Baſe; which multiply'd by 14, the Depth or Fall of the Water, and the Product is 11259.49, *&c.* which divided by 1728, the Quotient is 6.51 Feet; and ſo much is the ſolid Content requir'd.

CHAP. V.

Practical Queſtions in MEASURING.

Queſtion 1. IF a Pavement be 47 Feet 9 Inches long, and 18 Feet 6 Inches broad; I demand how many Yards are contained therein?

F.	I.		F. I.
47	9		47·75
18	6		18.5
376	0		23875
47			38200
23	10	6	4775
9	0	0	
4	6	0	9)883·375
883	4	6	98.1

Anſwer, 98 Yards 1 Foot.

Queſt. 2. There is a Room, whoſe Length is 21.5 Feet, and the Breadth 17.5 Feet, is to be pav'd with Stones, each 18 Inches ſquare; I demand how many ſuch Stones will pave it?

21.5 1.5
17.5 1.5

1075 75
1505 15
215

2.25 Area of one Stone.
2.25)376.25(167

1512
1625

50 Anſwer, 167 Stones.

Queſt. 3. There is a Room 109 Feet 9 Inches about, and 9 Feet 3 Inches high, which is all (except two Windows, each 6 Feet 6 Inches high, and 5 Feet 9 Inches broad) to be hung with Tapeſtry that is Ell-broad; I deſire to know how many Yards will hang the ſaid Room?

From the Content of the Room, ſubtract the Content of the Windows, and divide the Remainder by the Square Feet in a Yard of Tapeſtry.

```
3.75       109.75 Length.                        5.75
   3        92.5  Breadth.                       6.5
-----      -------                              -----
11.25       54875                                2875
           21950                                3450
          98775                                 ------
          -------                               37.375
        1015.1875 Content of the Room.               2
          74.75   Content of the Windows ſub.   ------
       ----------                               74.750
   11.25)940.4375(83.59
         --------
          4043
           6687
           10625
         --------
            500
```

Anſwer, 83.59 Yards.

Queſt. 4. If the Axis of a Globe be 27.5 Inches; I demand the Content, ſolid and ſuperficial.

```
     3.1416
       27.5
    -------
    157080
   219912
   62832
   -------
86.39.400 The Circumference.
```

86.394

86.394 The Circumference.
27.5 The Diameter.

431970
604758
172788

6)2375.8350 The ſuperficial Content.

395.9725 A ſixth Part.
27.5

19798625
27718075
7919450

10889.24375

Anſwer, { 6.3 Feet ſolid.
16.49 Feet ſuperficial. }

Queſt. 5. There is the Fruſtrum of a Globe, the Diameter of whoſe Baſe is 24 Inches, and the Altitude thereof is 19 Inches; what is the Content ſolid and ſuperficial?

Find the Superficies as is directed in Page 188, and 24 the Solidity by the firſt Theorem in Page 190.

24
24

96
48

576

.7854
576

47124
54978
39270

452.3904 } add.
314.16

766.5504 The Curve Superficies.
452.3904 The Baſe add.

1218.9408 { The whole ſuperficial Co
tent in Inches.

.7854
400

314.1600

20
20

400

12×12=144
3

432
100 The Square of the Alt. add.

532 The Sum.
10 Multiply by the Alt.

5320
5236 Multiply.

31920
15960
10640
26600

2785.5520 The Solidity in Inches.

Queſt. 6. If a Tree girt 18 Feet 6 Inches, and be 24 Feet long; how many Tons of Timber are contained in that Tree?

F.	I.			
4)18	6	The Girth.		
4	7	6		A 4th Part
4	7	6		
18	6	0		
2	8	4	6	
	2	3	9	
21	4	8	3	Here I multiply by 6 and by 4, because 6 times 4 is 24
			6	
128	4	1	6	
			4	
4|0)51|3	4	6	0	
12	33			

Anſwer, 12 Tons 33 Feet 4 Inches 6 Parts.

Note, That 40 Feet of Timber is a Ton, and 50 Feet a Load.

Note alſo, That 4 Feet broad, 4 Feet deep, and 8 Feet long, is a Chord of Fire-wood, that is, 128 cubical Feet.

Queſt. 7. There is a Cellar to be dug by the Floor, whoſe Length is 33 Feet 7 Inches, and the Breadth is 18 Feet 9 Inches, and its Depth is to be 5 Feet 9 Inches; I demand how many Floors of Earth are in that Cellar?

F.	I.		
33	7		The Length.
18	9		The Breadth.
264			
33			
16	9	6	
8	4	9	
9	0	0	
1	6	0	
629	8	3	
5	9	0	The Depth.
3148	5	3	
314	10	1	
157	5	0	
324)3620	8	4(11	
380			
56			

Anſwer, 11 Floors 56 Feet.

Note, That 18 Feet ſquare, and a Foot deep, is a Floor of Earth, that is, 324 ſolid Feet.

Queſt. 8. There is a Roof covered with Tiles, whoſe Depth on both Sides (with the uſual Allowance at the Eaves) is 35 Feet 6 Inches, and the Length 48 Feet 9 Inches; How many Squares of Tiling are contain'd therein?

F.	I.	
48	9	
35	6	
240		
144		
24	4	6
17	6	0
8	9	0
17|30	7	6

Anſwer, 17 Squares 30 Feet.

Queſt. 9. There is a Cone, whoſe Diameter at the Baſe is 42 Inches, and the perpendicular Height 94 Inches, and it is requir'd to cut off two ſolid Feet from the Top End thereof; I demand what Length upon the Perpendicular muſt be cut off?

```
       42    1728        94
       42       2        94
      ---    ----       ---
       84    3456       376
      168               846
     ----              ----
     1764              8836 Square
    .7854                94
    -----             -----
     7056             35344
    8820             79524
   14112              ------
  12348              830584 The Cube.
  ---------
  1385.4456
         94
  ---------
   55417824
 124690104
 -----------
3)130231.8864
 -----------
  4 410.6288
```

All the Solid Bodies are in triplicate Reaſon of their homologous Sides by *Euc.* 12, 12; 12, 18; and 11, 33: therefore it will be,

```
Solidity of the Cone.   Cub. Alt.   Solidity of 2 Feet.
  43410.6288        :    830584   : : 3456 :
                           3456
                        -------
                        4983504
                       4152920
                      3322336
                     2491752
                     ----------
434106288)2870498304(66124 The Cube of the
                                  Length.
    ....  ---------
          265860576
           5396803
           1055740
            187528
```

66124(40.43
64
2124000 Reſolvend.
12
48
492 Diviſor.
120
4800
48120 Diviſor.
64
1920
19200
1939264 Subtrahend.
184736000 Reſolvend.
1212
489648
4897692 Diviſor.
27
10908
1468944
147003507 Subtrahend.
37732493

Anſwer, The Length upon the Perpendicular muſt be 40.43 Inches. If it had been 3 Feet, the Length had been 46.29 Inches.

If two Feet were to be cut off from the Bottom, or greateſt End, then from 43410.6288 ſubtract 3456, and the Remainder is 39954.6288. Then ſay,

```
43410.6288 : 830584 : : 39954.6288
                             830584
                         ----------
                         1598185152
                        3196370304
                       1997731440
                     11986388640
                    3196370304
                    ---------------------    .  .
      13410.6288)33185675407.2192(764459(91.4
      .....      ---------------      729
                  279823524          -----
                   19359751          35459
                    1995500          -----
                     259075             27
                      42022            243
                     ------          -----
                       2952           2457
                                     -----
                                       271
                                     243
                                     -----
                                     24571
                                     -----
                                     10888000
                                     ---------
                                          273
                                       24843
                                       -------
                                       248703
```

Anſwer. It muſt be cut at 91.4 Inches from the Top, or 2.6 Inches from the Bottom.

Queſt. 10. If a Square Piece of Timber be 12 Feet long, and if the Side of the Square of the greater Baſe be 21 Inches, and the Side of the Square of the leſſer Baſe be 3 Inches; How far muſt I meaſure from the greater End, to cut off 5 ſolid Feet?

Firſt find the Length of the whole Pyramid, thus; the Difference between 21 and 3 is 18: Then,

Diff. Length. great. Length.
As 18 : 12 : : 21 : 14

So I find the whole Length of the Pyramid 14 Feet, or 168 Inches.

The ſolid Content of the whole Pyramid is 24696 Inches, and the ſolid Content of 5 Feet is 8640; which ſubtracted from 24696, there remains 16056 Inches. Then, the Cube of 168 (the Length) is 4741632. Then,

24696 : 4741632 : : 16056 :
3082752, whoſe Cube Root is 145.54; ſubtract this Root from 168 (the Length), and there remain 22.46 Inches, which is the Length of 5 ſolid Feet at the great End.

Queſt. 11. Three Men bought a Grinding-ſtone of 40 Inches Diameter, which coſt 20 Shillings; of which Sum the firſt Man paid 9 Shillings; the ſecond 6 Shillings, and the third 5 Shillings; I demand how much of the Stone each Man muſt grind down, proportionable to the Money he paid?

All Circles are in duplicate Reaſon of their Diameter, by *Euc.* 12, 2.

Square the Semidiameter, which makes 400. Then

s. Square. s.
20 : 400 : : 9 : 180

This 180 is the Square of the Semidiameter of the Circle belonging to the firſt Man.

And,

s. s.
And, 20 : 400 : : 6 : 120

This 120 is the Square of the Semidiameter of the Circle belonging to the ſecond.

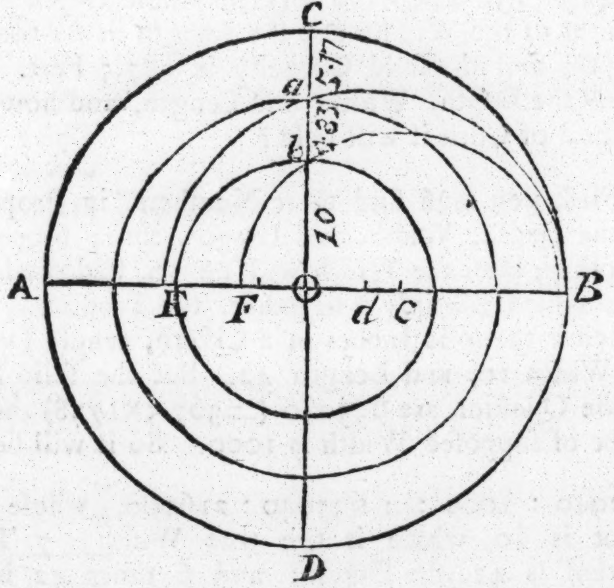
s. s.
And, 20 : 400 : : 5 : 100

This 100 is the Square of the Semidiameter of the Circle belonging to the third.

Then, from 400 (the Square of the Semidiameter of the Stone) ſubtract 180; and there remains 220, whoſe Square Root is 14.83 Inches; which ſubtracted from 20 Inches (the Semidiameter), there remain 5.17 Inches, which is the Breadth of the Ring, or Part of the Stone which muſt be ground down by the firſt.

Then, from 220 ſubtract 120, and there remains 100, whoſe Square Root is 10; ſubtract that from 14.83, and there remain 4.83 Inches, the Breadth of the Ring, or Part to be ground down by the ſecond Man. The third muſt grind down the Remainder, which is 10 Inches, the Square Root of 100.

This Queſtion may very eaſily and ſpeedily be perform'd Geometrically, as in the annexed Scheme.



Firſt, upon the Centre ⊙ ſtrike the Circle ABCD, and croſs it at right Angles with the two Diameters AB and CD: Then divide the Semidiameter A ⊙, (which ſuppoſe 20) in Proportion to 9 s. 6 s. and 5 s. (the ſeveral Sums paid by the three Men) by the Points E and F; ſo ſhall AE be 9, EF 6, and F ⊙ 5: Then divide EB into 2 equal Parts in d, and upon d, as a Centre, ſtrike the Semicircle EaB; and divide FB into 2 equal Parts in c, and upon c, as a Centre, with the Radius cF, ſtrike the Semicircle FbB: So have you the Semidiameter ⊙ C divided into three ſuch Parts as the Stone ought to be divided; and Circles, ſtruck thro' thoſe Points, will ſhew how much each Man muſt grind for his Share.

Queſt. 12. A Gard'ner he had an upright Cone,
Out of which ſhould be cut him a Rolling-ſtone,
The biggeſt that e'er it could make:
The Maſon he ſaid, That there was a Rule
For ſuch Sort of Work, but he had a thick Skull:
Now help him for Pity's Sake.

Anſwer, It muſt be cut at One-third Part of the Altitude.

Queſt. 13. There is a Ciſtern, whoſe Depth is ſeven Tenths of the Width, and the Length is ſix times the Depth, and the ſolid Capacity is 367.5 Feet. I demand the Depth, Width, and Length, and how many Buſhels of Corn it will hold?

Firſt, you muſt find three Numbers, in Proportion to the Depth, Width, and Length, thus; ſuppoſe the Depth 7, then the Width will be 10, and the Length 42; which multiply'd together, the Product is 2940, which is the ſolid Inches in a Ciſtern, whoſe Depth is 7, Width 10, and Length 42. But the ſolid Inches in the Queſtion are 635040 ($=367.5 \times 1728$) then the Cube of ſuppoſed Width is 1000. So it will be,

2940 : 1000 : : 635040 : 216000, whoſe Cube Root is 60, which is the true Width; 7 Tenths thereof is 42, the Depth; and 6 times 42 is 252 Inches, the Length; which three Numbers being multiply'd together, the Product will be 635040. If theſe ſolid Inches be divided by 2150.42, and the Quotient is 295 $\frac{66610}{215042}$ Buſhels, or 36 Quarters 7 Buſhels 1 Peck 4 Pints. And ſo much will the Ciſtern hold.

Queſt. 14. Suppoſe, Sir, a Buſhel be exactly round,
Whoſe Depth being meaſur'd, 8 Inches is found,
If the Breadth 18 Inches and half you diſcover,
This Buſhel is legal all England over.
But a Workman would make one of another Frame;
Sev'n Inch and a half muſt be the Depth of the ſame:
Now, Sir, of what Length muſt the Diameter be,
That it may with the former in Meaſure agree?

18.5

18.5

925

1480

185

342.25 The Square.

.7854

136900

171125

273800

239575

268.803150

8

2150.425200 The ſolid Inches in a Buſhel.

7.5)2150.4252(286.72336

. . .

650

504

542

175

252

270

450

. . .

.7854)286.72336

51103
39793
5236

(365 0666(19.107
1

29)265
261

381)406
381

38207)256600

Anſwer, The Diameter muſt be 19.107 Inches, if the Depth be 7.5 Inches.

Queſt. 15. In the Midſt of a Meadow well ſtored with Graſs,
I took juſt an Acre to tether my Aſs;
How long muſt the Cord be, that feeding all round,
He mayn't graze leſs nor more than his Acre of Ground?

By Problem 10. Section IX. Chap. 1. find the Diameter of a Circle containing an Acre; half that will be the Length of the Cord.

The Work.

660 Feet, the Length of an Acre.
66 Feet, the Breadth of an Acre.

3960
3960

43560 The Square Feet in an Acre.

```
As 1 : 1.2132 : : 43560
                  43560
                 ------
                 763920
                63660
               38196
              50928
              -----
              . . .
              55460.5920(235.5 Diameter.
              4         ------
              ----      117.75 Half.
           43)154
              129
              ----
          465)2560
              2325
              -----
         4705)23559
              23525
              -----
                 34
```

Anſwer, The Chord muſt be 117 Feet and 9 Inches.

Queſt. 16. A Malſter has a Kiln, that is 16 Feet 6 Inches ſquare; but he is minded to pull it down, and build a new one, that may be big-enough to dry three times as much at a time as the old one will do; I demand how much ſquare the new one muſt be?

```
        16.5
        16.5
       -----
         825
        990
       165
     -------
      272.25 The Area of the old one.
           3
      ------
      .  .  .
      816.75(28.57
      4
      ----
   48)416
      384
      ----
  565)3275
      2825
      -----
 5707)45000
      39949
      -----
       5051
```

Anſwer, The Side of the new one muſt be 28 Feet, and near 7 Inches.

Queſt. 16. If a round Ciſtern be 26.3 Inches Diameter, and 52.5 Inches deep; How many Inches Diameter muſt a Ciſtern be to hold twice the Quantity, the Depth being the ſame? and how many Ale-Gallons will each Ciſtern hold?

26.3
26.3

789
1578
526

691.69 The Square.
2

1383.38(37.19
9

67)483
469

741)1438
741

7429)69700
66861

[2839

The Diameter of the greater is 37.19 Inches.

691.69 The Squ. of the leſſ. Ciſtern's Dia.
.7854

276676
345845
553352
484183

543.253326 Area of the Baſe.
52.5

2716266630
1086506652
2716266630

28520.7996150 Solid Content in Inches.

282)28520.799(101.137 Gallons.

320
387
1059
2139

165

Note, That 282 ſolid Inches is an Ale or Beer Gallon, and 231 a Wine Gallon.

And 359.05 is the Square of the Diameter of a Circle that will hold a Gallon of Ale at an Inch deep, and 294.12 for Wine.

You may find the Content in Gallons, thus: Divide the Square of the Diameter by 359.05, and multiply the Quotient by the Depth.

359.05)1383.38 (3.853
52.5
306230
18990
1037
19265
7706
19265

The Content of the greater 202.2825 Gallons.

Queſt. 18. If the Diameter of a Caſk at the Bung be 32 Inches, and at the Head 25 Inches, and the Length 40 Inches; How many Ale Gallons are contain'd therein?

25 32 359
25 32 3
125 64 1077
50 96
625 1024 Square of the Bung Diameter.
1024 The ſame.
625 Square of the Head Diameter.

1077)2673 (2.48
40
5190
8820 99.20
204

Anſwer, 99.2 Gallons.

Otherwiſe you may find a mean Diameter, and work by Scale and Compaſſes, thus: Subtract 25 from 32, and there remains 7, which multiply'd by .7, the Product is 4.9, which added to 25, the Sum is 29.9. Then extend the Compaſſes from 18.95 to 29.9, that Extent, turn'd twice from 40 (the Length) will fall upon 99.6 Gallons; ſomething more than before.

Queſt. 19. There is a Stone 20 Inches long, 15 Inches broad, and 8 Inches thick, which weighs 217 Pounds; I demand the Length, Breadth, and Thickneſs of another of the ſame Kind and Shape, which weighs 1000 Pounds?

The Cube of 20 (the Length) is 8000. Then (by *Euc.* 11.33).

217 : 8000 : : 1000 : 36870.645, whoſe Cube Root is 33.28 Inches, the Length of the Stone weighing 1000 Pounds. Then ſay,

20 : 33.28 : : 15 : 24.96
20 : 33.28 : : 8 : 13.312

$$\text{Anſwer, }\left\{\begin{array}{ll}\text{The Length} & 33.28\\ \text{The Breadth} & 24.96\\ \text{The Thickneſs} & 13.312\end{array}\right\}\text{ Inches.}$$

Queſt. 20. If an iron Bullet, whoſe Diameter is 4 Inches, weighs 9 Pounds; What will be the Weight of another Bullet (of the ſame Metal) whoſe Diameter is 9 Inches?

The Cube of 4 is 64, and the Cube of 9 is 729. Then (by *Euc.* 12.18.),

$$64 : \overset{lb.}{9} : : 729 : \overset{lb.}{102.515}$$

Anſwer, It weighs 102 *lb.* 8 *oz.* 4 *dr.* *fere.*

Queſt. 21. There is a Square Pyramid of Marble; each Side of its Baſe is 5 Inches, and the Height thereof 15 Inches, and its Weight is 12 Pounds and a Quarter; I demand the Weight of another like Square Pyramid, each Side of whoſe Baſe is 30 Inches?

The Cube of 5 is 125, and the Cube of 30 is 27000. Then (by *Euc.* 12. 12.)

lb. *lb.*
125 : 12.25 : : 27000 : 2646
Anſwer, The Weight is 2646 Pounds.

Queſt. 22. There is a Ball or Globe of Marble, whoſe Diameter is 6 Inches, and its Weight 11 Pounds; What will be the Diameter of another Globe of the ſame Marble, that weighs 500 Pounds?

The Cube of 6 is 216. Then,
lb. *lb.*
11 : 216 : : 500 : 981.1818.
Whoſe Cube Root is 21.4 Inches, the Diameter ſought.

Queſt. 23. There is a Fruſtum of a Pyramid, whoſe Baſes are regular Octagons; each Side of the greater Baſe is 21 Inches, and each Side of the leſſer Baſe is 9 Inches, and its Length is 15 Feet; I demand, how many ſolid Feet are contained therein?

```
 4.8284 The tabular Number, Page 88.
    237 The Square of a mean Side.
 ------
 337988        21       12
 144852         9       12
 96568        ---      ---
 ------       189   3)144
1144.3308      48      ---
       15     ---       48
---------     237
 57216540
11443308
---------
144)17164.9620(119.2
  ----------
  276
  1324
   289
  ---
    1
```

Anſwer, 119.2 ſolid Feet.

Queſt. 24. There is a Fruſtum of a Cone, the Diameter of the greater Baſe is 36 Inches, and the Diameter of the leſſer Baſe is 20 Inches, and the Length or Height is 215 Inches; I demand the Length and ſolid Content of the whole Cone, and alſo the ſolid Content of the given Fruſtum?

Firſt, Find the Length of the whole Cone, thus:

```
   From  36
   Subtr. 20
          --
          16 : 215 : : 36 : 483.75
```

So the Length of the whole Cone is 483¾ Inches.

Then find the Content of the whole Cone.

36
36

216
108

1296
.7854

5184
6480
10368
9072

1017.8784 Area Baſe.

1017.8784
52.161

10178784
6107270
101788
20357
5089

1728)16413 2.88(94.98 Feet.

8612
17008
14568

744

Thus I find the Solidity of the whole Cone 94.98 Feet.

Then find the ſolid Content of the Top-part that is wanting.

.7854

.7854	The Area of Unity.
400	The Square of 20.
3)314.1600	Area of the leſſer Baſe.
104.72	A third Part.
268.75	Altitude of the Top-part.

52360

73304

83776

62832

20944

1728)28143.5000(16.28 Feet.

10863

4955

14990

1166

	Feet.
Content of the Whole	94.98
Content of the Top-piece	16.28
Content of the Fruſtum	78.7

Queſt. 25. If the Top-part of a Cone contains 26171 ſolid Inches, and 200 Inches its Length, and the lower Fruſtum thereof contains 159610 ſolid Inches; I demand the Length of the whole Cone, and the Diameter of each Baſe?

200	159610 } add.
200	26171 }
40000	185781 The Sum.
200	
8000000	

26171

26171 : 8000000 : : 185781 : 56789881, whoſe Cube Root is 384.3 Inches, the Length of the whole Cone.

Then find the Diameter of the leſſer Baſe, thus:

200)26171

130.855
3

392.565 Area of the leſſer Baſe.

Then by Prob. 10. Sect. IX. Chap. 1.

1 : 1.2732 : : 392.565
1.2732

785130
1177695
2747955
785130
392565

499.8137580(22.35
4

42)99
84

443)1581
1329

4465)25237
22325

2912

Leſſer Leng. Leſſ. Diam. Gr. Leng. Gr. Diam.
Again, 200 : 22.35 : : 384.3 : 42.94.

		Inches.
Anſwer,	The Length of the whole Cone	384.3
	The Diameter of the greater Baſe	42.94
	The Diameter of the leſſer Baſe	22.35

Queſt. 26. There is a Fruſtum of a Cone, whoſe ſolid Content is 20 Feet, and its Length 12 Feet; and the greater Diameter bears ſuch Proportion to the leſſer as 5 to 2; I demand the Diameters?

5 × 5 = 25
2 × 2 = 4
5 × 2 = 10
The Sum 39

3)12
4)20(5 Feet.

Theſe 5 Feet are the Triple of a mean Area.

Then, 1 : 1.27324 : : 5 : 6.3662

So the Triple Square of a mean Diameter is 6.3662.

Then, 39 : 6.3662 : : 25 : 4.080897

This 4.080897 is the Square of the greater Diameter, whoſe Square Root is 2.020123 Feet, which is 24.24147 Inches. Then,

5 : 24.24147 : : 2 : 9.69659

So the greater Diameter is 24.24147, and the leſſer Diameter is 9.69659 Inches.

Queſt. 27. There is a Room of Wainſcot 129 Feet 6 Inches in Circumference, and 16 Feet 9 Inches high (being girt over the Mouldings); there are two Windows, each 7 Feet 3 Inches high, and the Breadth of each, from Cheek to Cheek, 5 Feet 6 Inches; the Breadth of the Shutters of each is 4 Feet 6 Inches; the Cheek-boards and Top and Bottom-boards of each

Window, taken together, is 24 Feet 6 Inches, and their Breadth 1 Foot 9 Inches; the Door-caſe 7 Feet high, and 3 Feet 6 Inches wide; the Door 3 Feet 3 Inches wide; I demand how many Yards of Wainſcot are contain'd in that Room?

F.	I.	
129	6	
16	9	
782	0	
129		
64	9	
32	4	6
2169	1	6

F.	I.		
3	3		
7	0		
22	9		
11	4	6	Half.
34	1	6	

F.	I.	
7	3	
5	6	
36	3	
3	7	6
39	10	6
		2
79	9	0

F.	I.		
7	3		
4	6		
29	0		
3	7	6	
32	7	6	
16	3	9	Half.
48	11	3	
		2	
97	10	6	

F.	I.	
24	6	
1	9	
4	6	
18	4	6
42	10	6
		2
85	9	0

F.	I.	
3	6	
7	0	
24	6	} add.
79	9	}
104	3	

The

The Content of the Room	2169	1	6
The Shutters, at Work and half	97	10	6
The Door, at Work and half	34	1	6
The Cheek-boards, *&c.*	85	9	0
The Sum	2386	10	6
The Window-lights and Door-caſe deduct	104	3	0
	9)2282	7	6
	253	5	

Anſwer, 253 Yards 5 Feet.

Queſt. 28. There is a Wall which contains 18225 Cube Feet, and the Height is 5 times the Breadth, and the Length 8 times the Height; What is the Length, Breadth, and Height?

Suppoſe the Breadth 2, then the Height muſt be 10, and the Length 80; which three Numbers multiply'd together, the Product will be 1600, and the Cube of 2 is 8; then ſay,

1600 : 8 : : 18225 : 91.125.

Then the Cube Root of 91.125 is 4.5, which is the Breadth; then 5 times 4.5 is 22.5 the Height; and 8 times 22.5 is 180, the Length.

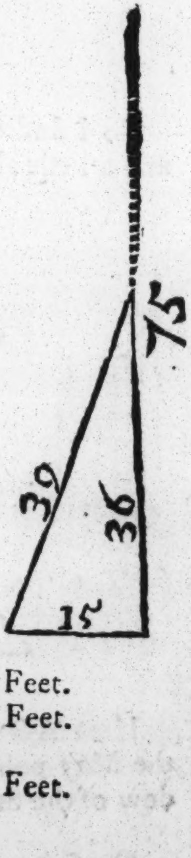
Queſt. 29. There is a May-pole, whoſe Top end was broken off by a Blaſt of Wind, and the Top end, in falling, ſtruck the Ground at 15 Feet Diſtance from the Foot of the May-pole, the broken Piece was 39 Feet; Now I demand the Length of the May-pole?

By *Eucl.* 1.47. the Square of the Hypothenuſe of a right-angled Triangle is equal to the Sum of the Squares of the Baſe and Perpendicular.

Therefore, from the Square of 39 ſubtract the Square of 15, the Square Root of the Remainder is the Piece ſtanding; to which add the Piece broken off, and you have the whole Length.

39	15
39	15
351	75
117	15
1521	225
225	

1296(36
9
66)396
396
. . .



The Piece ſtanding is	36 Feet.
The Piece broken off is	39 Feet.
The whole Length	75 Feet.

Queſtion 50.

A May-pole there was, whoſe Height I would know:
The Sun ſhining clear, ſtrait to work I did go:
The Length of the Shadow, upon level Ground,
Juſt ſixty-five Feet, when meaſur'd, I found;
A Staff I had there, juſt five Feet in Length;
The Length of its Shadow was four Feet one Tenth:
How high was the May-pole, I gladly would know?
And it is the Thing you're deſir'd to ſhow.

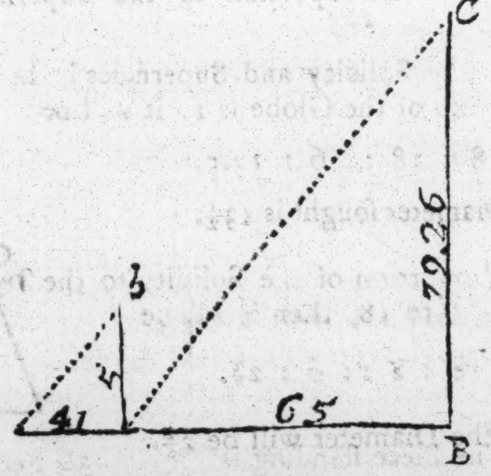
By *Eucl.* 6. 4.

a A : A b : : A B : B C.

That is,

4.1 : 5 : : 65 : 79.26.

So I find the Height of the May-pole to be 75 Feet, and a little above 3 Inches.



Here A B repreſents the Length of the Shadow of the May-pole, and B C the May-pole; a A the Shadow of the Staff, and A b the Staff.

Queſt. 31. What will be the Diameter of a Globe, when the Solidity and ſuperficial Content thereof are equal?

If the Diameter be 1, the Solidity will be .5236, and the Superficies will be 3.1416; that is, as 1 to 6. And to find the ſuperficial Content, we muſt multiply 3.1416 by the Square of the Axis or Diameter, and the Product is the ſuperficial Content. And for the Solidity, multiply .5236 by the Cube of the Axis, the Product is the ſolid Content; therefore, becauſe .5236 is a ſixth Part of 3.1416, we muſt take 6 for the Diameter ſought. For if 3.1416 be multiply'd by the Square of 6, *viz.* by 36, the Product will be 113.0976; and if .5236 be multiply'd by the Cube of 6, *viz.* by 216, the Product is likewiſe 113.0976, the Solidity equal to the Superficies.

Therefore, 6 is the true Anſwer.

Queſt. 32. What will the Axis of a Globe be, when the Solidity is in Proportion to the Superficies, as 18 to 8?

Becauſe the Solidity and Superficies is as 1 to 6; when the Axis of the Globe is 1, it will be

$$8 : 18 :: 6 : 13.5.$$

So the Diameter ſought is $13\frac{1}{2}$.

If the Proportion of the Solidity to the Superficies had been as 8 to 18, then it will be

$$18 : 8 :: 6 : 2\frac{2}{5}.$$

So then the Diameter will be $2\frac{2}{5}$.

The Reaſon of theſe Operations, both in this and the laſt Queſtion, is from Algebra.

Queſt. 33. There are three Grenado-ſhells, of ſuch Capacity, that the ſecond Shell will juſt lie in the Concavity of the firſt, and the third in the Concavity of the ſecond. The Solidity of the Metal of the firſt Shell is equal to its Concavity, and the Solidity of the Metal to the ſecond, to the Concavity, is as 7 to 5; and the Solidity of the third, or leaſt Shell's Metal, to its Concavity, is as 9 to 4. Now, ſuppoſing the Diameter of the firſt, or greateſt Shell, to be 16

Inches, and allowing every ſolid Inch of Iron to weigh 4 Ounces; I demand the Diameter of the two leſſer Shells; and the Thickneſs and Solidity of Metal of every Shell; and alſo the Weight of every Shell?

The Cube of 16 is 4096; then,

1 : .5236 : : 4096 2144.6656.

The half thereof is 1072.3328, which is the Solidity of the Metal of the greater Shell, as alſo the Concavity.

.5236 : 1 : : 1072.3328 : 2048.

The Cube Root of 2048 is 12.699, which is the Diameter of the ſecond Shell.

The Sum of 7 and 5 is 12; then,

12 : 5 : : 1072.3328 : 446.805.

This 446.805 is the ſolid Content of the Concavity of the ſecond.

.5236 : 1 : : 446.805 : 853.333.

The Cube Root of 853.333 is 9.485, the Diameter of the leaſt Shell.

The Sum of 9 and 4 is 13; then,

13 : 4 : : 446.805 : 137.47846.

This 137.47846 is the ſolid Content of the Concavity of the third.

.5236 : 1 : : 137.47846 : 262.5639.

The Cube Root of 262.5639 is 6.4034, the Diameter of the leaſt Shell's Concavity.

From 16 The Diameter of the greateſt,
Subtr. 12.699 The Diameter of the ſecond.

Rem. 3.301

Half is = 1.65 The Thickneſs of Metal of the greateſt

From 12.699 The Diameter of the ſecond,
Subtr. 9.485 The Diameter of the leaſt.

Rem. 3.214

Half is=1.607 The Thickneſs of Metal of the ſecond.

From 9.485 The Diameter of the leaſt,
Subtr. 6.403 The Diameter of the Concavity.

Rem. 3.082

Half is=1.521 The Thickneſs of Metal of the leaſt.

The Metal of the greateſt is 1072.33 ſolid Inches; which divide by 4 (becauſe every ſolid Inch is a Quarter of a Pound) the Quotient is 268.08 Pounds.

The Metal of the ſecond is 625.52 ſolid Inches; which divided by 4, the Qnotient is 156.38 Pounds, the Weight of the ſecond.

The Metal of the leaſt Shell is 309.32 ſolid Inches; which divided by 4, the Quotient is 77.33 Poun[illegible]; the Weight of the leaſt.

The Diam. of the { ſecond Shell 12.699 } Inches.
{ leaſt Shell 9.485 }

The Thickneſs of the Metal of the { greateſt 1.65 } Inches.
{ ſecond 1.607 }
{ leaſt 1.541 }

The Weight { greateſt 268.08 } Pounds.
{ ſecond 156.38 }
{ leaſt 77.33 }

282
36

1692
846

101.52

BRITISH MUSEUM
30 AUG 69

A SHORT APPENDIX.

§ I. *Of* GAUGING.

I SHALL not here give the whole Art of Gauging (there being ſeveral Books of that Art already in Print, writ by better Hands); but ſhall only lay down ſome ſhort practical Rules, whereby any Artificer, or others, may find the Quantity of Liquor in any Veſſel upon Occaſion.

◆

PROBLEM I.

To find the ſeveral Multipliers, Diviſors, and Gauge-points belonging to the ſeveral Meaſures now uſed in England.

282)1.0000(.003546 Multiplier for Ale Gallons.
231)1.0000(.004329 Multiplier for Wine Gallons.
268.8)1.000(.0037202 Multiplier for Corn Gallons.
2150,42)1.000(.00046502 Multipl. for Corn Buſhels.

So, if the ſolid Inches in any Veſſel be multiply'd y the ſaid Multipliers, the Product will be Gallons in their reſpective Meaſures; or dividing by the Diviſors 282, 231, or 268.8, the Quotient will likewiſe be Gallons.

Note, That 282 ſolid Inches is a Gallon of Ale or Beer-meaſure; 231 ſolid Inches is a Gallon of Wine-meaſure; 268.8 ſolid Inches is a Gallon, and 2150.42 ſolid Inches is a Buſhel of Corn-meaſure.

For circular Area's, the following Multipliers and Diviſors are to be uſed.

282).785398(.002785 Multipliers for Ale Gallons.
231).785398(.003399 Multiplier for Wine Gallons.
.785398(282.(359.05 Diviſor for Ale Gallons.
.785398(231.(294.12 Diviſor for Wine Gallons.
.785398)2150.42(2738 Diviſor for Corn Buſhels.

The Square Root of the Diviſor is the Gauge-point.

The Gauge-point for Squares in	Ale-meaſure, is	16.79
	Wine meaſure, is	15.19
	Malt-buſhel, is	46.36
The Gauge-point for circular Figures in	Ale meaſure, is	18.95
	Wine-meaſure, is	17.15
	Malt-buſhel, is	52.32

PROBLEM II.

To find the Area in Ale or Wine Gallons, of any rectilineal plain Figure, whether Triangular, Quadrangular, or Multangular.

TO reſolve this Problem, you muſt, by Chap. I. Part II. find the Area in Inches, and then bring it to Gallons, by dividing that Area in Inches by the proper Diviſor; *viz.* by 282 for Ale, or by 231 for Wine; or elſe by Multiplication, by .003.546 for Ale, or by .004329 for Wine; and the Quotient or Product will be the Area.

Example. Suppoſe a Back, or Cooler, in the Form of a Parallelogram; or long Square, 250 Inches in Length, and 84.5, Inches in Breadth; What is the Area in Ale or Wine Gallons?

Multiply 250 by 84.5, and the Product is 21125, the Area in Inches, which divide by 282, and the Quotient is 74.9 Gallons of Ale; or multiply'd by .003546, the Product is 74.20928 Gallons, nearly the ſame; and if 21125 be divided by 231, or multiply'd by .004329, it will give 91.44 Gallons of Wine.

By Scale and Compaſſes.

Extend the Compaſſes from 282 to 250, that Extent will reach from 84.5 to 74.9: And,

Extend from 231 to 250, that Extent will reach from 84.5 to 91.45.

Note, The Area's of all ſuperficies are always to be underſtood to be 1 Inch deep; otherwiſe it could not be ſaid, that the Area of ſuch a Parallelogram, Circle, *&c.* is ſo many Gallons.

Having found the Area of a Back, or Cooler, the next Thing will be to find out the true Dipping or Gauging-place in that Back, that ſo the true Quantity of Worts may be computed at any Depth, which may be thus done.

1. When the Bottom of the Back is cover'd all over (of any Depth) with Worts, or other Liquor, then dip it in eight or ten ſeveral Places (more or leſs, according to the Largeneſs of the Back), as remote and equally diſtant from each other as you can well do, noting down the wet Inches and decimal Parts of every Dip.

2. Divide the Sum of all thoſe Dips by the Number of Places you dipp'd in, and the Quotient will be the mean Wet of all thoſe Dips.

3. Laſtly, find out ſuch a Place by the Side of the Back (if you can) that juſt wets the ſame with that mean Dip, and make a Notch or Mark there for the true and conſtant Dipping-place of that Back.

Then if any Quantity of Worts (which covers the whole Back) be deeped or gauged at that Place, and the wet Inches ſo taken be multiply'd into the Area of the Back in Gallons, the Product will ſhew how many Gallons of Worts are in that Back at that Time, provided the Sides of the Back do ſtand at Right-angles with the Bottom.

* * *

PROBLEM III.

The Diameter of a Circle being given in Inches, to find the Area thereof in Ale or Wine Gallons.

IF the Square of the Diameter be multiply'd by .002785 for Ale, or by .003399 for Wine; or if it be divided by 359.05 for Ale, or by 294.12 for Wine, the Products or Quotients will be the reſpective Ale or Wine Gallons.

Example. Suppoſe the Diameter of a Circle be 32.6 Inches; What will be the Area in Ale or Wine Gallons?

The Square of 32.6 is 1062.76.

Then 359.05)1062.76(2.9599 Area in Ale Gallons.
And 294.12)1062.76(3.6133 Area in Wine Gallons
Or 1062.76×.002785=2.9598 Ale Gallons,
And 1062.76×.003399=3.6133 Wine Gallons.

By Scale and Compaſſes.

Extend the Compaſſes from 18.95 (the Gauge-point for Ale) to 32.6 (the Diameter), that Extent will reach from 1 to a 4th Number, and from that 4th to 2.9599 Gallons. Or, extend the Compaſſes from 1 to 32.6, that Extent, turn'd twice over from .002785, will at laſt fall upon 2.9599.

For Wine extend from 17.15 (the Gauge-point for Wine) that Extent, turn'd twice over from 1, will at laſt fall upon 3.6133 Gallons.

Or thus: Extend from 1 to 32.6, that Extent will reach from .003299, being turn'd twice over, to 3.6133 Wine Gallons.

PROBLEM IV.

The Tranſverſe (or longeſt Diameter) and the Conjugate (or ſhorteſt Diameter) of an Ellipſis (or Oval) being given, to find its Area in Ale or Wine Gallons.

IF the Rectangle, or Product of the two Diameters, that is, of the Length and Breadth of the Oval, be divided by 359.05, or multiply'd by .002785 for Ale, or divided by 294.12, or multiply'd by .003399 for Wine, the Quotient or Product will be the Ale or Wine Gallons requir'd.

Example. Suppoſe the longeſt Diameter be 81.4 Inches, and the ſhorteſt Diameter be 54.6 Inches; What will be the Area of that Oval?

Multiply 81.4 by 54.6, and the Product is 4444.44; then

359.05)4444.44(12.38 Area in Ale Gallons.
294.12)4444.44(15.11 Area in Wine Gallons.
Or 4444.44×.002785=12.38 Ale Gallons.
And 4444.44×.003399=15.11 Wine Gallons.

By Scale and Compaſſes.

Firſt, find a mean Proportional between 81.4 and 54.6, by dividing the Diſtance between them into two equal Parts, and the middle Point will be at 66.6, which is the mean Proportional (that is, the Diameter of a Circle equal to the Oval). Then extend the Compaſſes from 18.95 (the Gauge-point for Ale) to 66.6, that Extent, turn'd twice over from 1, will at laſt fall upon 12.38, Ale Gallons: And extended from 17.15 (the Gauge-point for Wine) to 66.6, that Extent, turn'd twice over from 1, will reach at laſt to 15.11 Wine Gallons.

PROBLEM V.

To find the Content in Ale or Wine Gallons of any Priſm, what Form ſoever its Baſe is of.

FIRST, find its ſolid Content in Inches (by Sect. 1, 2, 3, of Chap. II. Part II.); then divide that Content in Inches by 282 for Ale, or by 231 for Wine; the reſpective Quotients will be the Content in Wine or Ale Gallons.

Otherwiſe, you may find the Content of a Priſm by finding the Area of its Baſe in Gallons (by Problem II. of this Appendix) and multiply that Area by the Tun's Height, or Depth within, the Product will be its Content in Gallons.

Example. Suppoſe a Tun, whoſe Baſe is a Parallelogram right-angled, its Length being 49.3 Inches, its Breadth 36.5 Inches, and the Depth of the Tun is 42.6 Inches; the Content in Ale and Wine Gallons is required.

The Length, Breadth, and Depth, being multiply'd continally, the Product is 76656.57; which divided by 282, the Quotient is 271.83 Ale Gallons: And divided by 231, the Quotient is 331.84 Wine Gallons: And by dividing by 2150.4, ſuch a Ciſtern will be found to hold 35.65 Buſhels of Corn.

By Scale and Compaſſes.

Extend the Compaſſes from 282 to 36.5, the Breadth of the Baſe, that Extent will reach from 49.3, its Length, to 6.38 Ale Gallons, the Area of the Baſe; then extend from 1 to 42.6, the Depth, that Extent will reach from 6.38, the Area of the Baſe, to 271.8 Gallons, the Content.

PROBLEM VI.

To find the Content of a Tun, whoſe Baſes are alike and parallel, but unequal, being the Fruſtum of a Pyramid.

FIND the Area of each Baſe, and a mean Proportional between them, and multiply the Sum of thoſe three by one third Part of the Depth or Height, and the Product is the Content.

Example. Suppoſe a Tun, whoſe Baſes are Parallelograms; the Length of the greater is 100 Inches, and its Breadth 70 Inches; the Length of the leſſer Baſe 80, and its Breadth 56; and the Depth of the Tun 42 Inches, the Content in Ale an Wine Gallons is requir'd.

Multiply 100 by 70, the Product is 7000, the Area of the greater Baſe; and 80 multiply'd by 56, the Product is 4480, the Area of the leſſer Baſe; then multiply the two Areas into each other; and the Product is 31360000, whoſe Square Root is 5600 a geometrical mean Proportional.

The greater Area	7000	add.
The leſſer Area	4480	
The mean Proportional	5600	
	17080	
A Third of the Depth	14	
	68320	
	17080	

282)239120(847.94 A. G.

231)239120(1035.15 W. G.

PROBLEM VII.

To find the Content of a Tun, whoſe Baſes are parallel and circular, being the Fruſtum of a Cone.

YOU may find the Content as in the laſt Problem, by multiplying the Sum of the Areas of the two Baſes, and a mean Proportional, by one third Part of the Depth.

But it will be a ſhorter Way to find the Area of a mean Circle in Gallons, and multiply that by the Depth, thus: To the Rectangle of the greater and leſſer Diameters add one third Part of the Square of the Difference of the Diameters; that Sum is the Square of a mean Diameter, which, divided by 359.05 for Ale, or by 294.12 for Wine, gives the Area of a mean Circle in Ale or Wine Gallons, which, multiply'd by the Depth, gives the Content.

Example. Suppoſe the greater Diameter 80 Inches, and the leſſer Diameter 71 Inches, and the Depth 34 Inches, the Content in Ale or Wine Gallons is requir'd.

Multiply 80 by 71, and the Product is 5680; to which add 27 (a third Part of the Square of the Difference of the Diameters) and the Sum is 5707, which is the Square of a mean Diameter; which divide by 359.05, and the Quotient will be 15.895 Gallons the Area; which multiply by 34 (the Depth), and the Product will be 540.43 Gallons the Content.

By Scale and Compaſſes.

Add the two Diameters together, and take half the Sum, which is 75.5, which take ſor a mean Diameter (though it is not exact, yet it will be near enough the Truth, if the Difference between the Diameters be not great); extend the Compaſſes from 18.95 (the Gauge-point for Ale) to 75.5, the mean Diameter; that Extent will reach from that 34 (the Depth) to a 4th Number, and from that to 540.4 Gallons, the Content.

And if you extend the Compaſſes from 17.15 (the Gauge-point for Wine) to 75.5, that Extent will reach from 34, twice turn'd over, 659.7 Gallons of Wine.

The Method uſed by the Gaugers for all ſuch Tuns is to take the Diameter in the Middle of every 10 Inches, that is, at five Inches from the Bottom, and at 15, and at 25, *&c.*

Then they find the Area to every one of theſe Diameters, and enter them in their Books. Then, when they ſurvey, they take the wet Inches and Parts that the Liquor in the Tun is in Depth, and every ten Inches they take the reſpective Areas, and remove the ſeparating Point one Place toward the Right-hand; and for what odd Inches of the Depth above the even Tens, they multiply the next Area by them, and ſo add all the ſeveral Products together, and the Total will be the Gallons of Liquor in the Tun.

Example. Suppoſe the Diameter at 5 Inches from the Bottom be 64 Inches, and at 15 Inches from the Bottom 67 Inches, and at 25 Inches 70 Inches, and at 35 Inches from the Bottom, the Diameter is 73 Inches. Now the Area anſwering to 64 Inches is 11 4078 Gallons; and to 67 Inches, is 12.5023 Gallons; and the Area to 70 Inches, is 13.647 Gallons; and to 73, is 14.8418 Gallons: Then, ſuppoſing the Depth of the Liquor in the ſaid Tun be found to be 3.6 Inches: Now, to caſt up this Gauge, firſt, in the Area anſwering to 64 Inches, being multiply'd by 10, that is by removing the ſeparating Point a Place towards the Right-hand, it will be 114.078 Gallons; and the next will be 125.023; and the next 136.47 Gallons. Now theſe three will be the Content to 30 Inches deep. Then, to find the Content of the 3 6 Inches, multiply the next Area 14.8418 by 3.6, and the Product is 53.4305: Add all together, and the Sum is the whole Quantity of Liquor in the Tun.

The Content at 10 Inches deep	114.078
The Content at the next 10 Inches	125.023
The Content at the next 10 Inches	136.470
The Content of the 3.6 Inches	53.430
The whole Quantity of Liquor in the Tun	429.001

Problem VIII.

To find the Drip or Full of a Tun.

SUppoſe the Tun laſt mention'd was ſo placed, that when the Bottom is but juſt covered on one Side, the Liquor is 4 Inches deep on the Side oppoſite: How much muſt be allow'd for the Fall of this Tun? That is, How much Liquor is there in the Tun?

The Diameter in the Middle of 4 Inches from the Bottom, is 61.6 Inches; and the Area anſwering thereunto is 10.568; which multiply'd by 2 (that is, half 4), the Product is 21.136 Gallons; and ſo much Liquor will juſt cover the Bottom.

But, ſuppoſe it was ſet ſo much on one Side, as to be 30 Inches deep on one Side, when the Liquor on the oppoſite Side juſt cuts between the Bottoms and Staves; How much Liquor will there be in the Tun?

Square the Bottom Diameter, and multiply that Square by the Top Diameter, and divide the laſt Product by the Sum of the Diameters, and to the Quotient add the Square of the Bottom Diameter, and divide the Sum by 1077.15 for Ale, or by 882.36 for Wine; multiply the Quotient by the Depth, the Product is the Content.

The Bottom Diameter of the fore-mention'd Tun is 61 Inches; and the Diameter, at 30 Inches from the Bottom, is 71.5 Inches; the Square of 61 is 3721; which multiply'd by 71.5, the Product is 266051.5; this divided by 132.5 (the Sum of the Diameters) the Quotient is 2007.936; to which add 3721 (the Square of 61), and the Sum will be 5728.936; this divided by 1077.15, the Quotient is 5.3186; which multiply'd by 30 the Depth, the Product is 159.558 the Gallons of Liquor in the Tun.

When the Fruſtum of a Cone or Pyramid is cut, by a Diagonal Plain, through the Extremities of the Diameters, as the Liquor in the Tun repreſents, ſuch Solid is called a Hoof (*Vide Ward*'s Young Mathematician's Guide, *Page* 414).

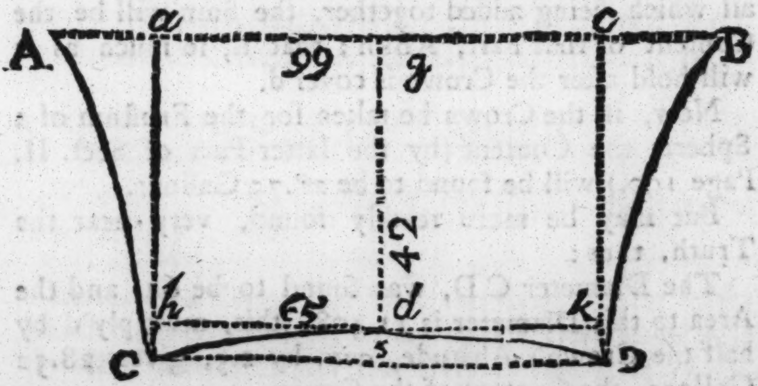
If it be the Hoof of a Square Fruſtum, inſtead of dividing by 1077.15, divide by 846 for Ale, or by 693 for Wine. All the reſt of the Work is the ſame.

PROBLEM IX.

To Gauge a Copper.

LET ABCD be a ſmall Copper to be gaug'd.

Take a ſmall Cord or Packthread, make one End faſt at A, and extend the other to the oppoſite Side of the Copper at B, where make it faſt, or cauſe ſome Perſon to hold it very ſtrait; then ſet one End of the Inſtrument in the Bottom of the Copper at C, and move it to and fro, till you find the neareſt Diſtance to the Thread (as at a): This Diſtance, a C, is the Depth of the Copper, which ſuppoſe to be 47 Inches.



In like manner, ſet the End of the Rule upon the Top of the Crown at d, and take the neareſt Diſtance to the Thread, as d g, which ſuppoſe 42 Inches; this ſubtracted from a C, 47, the Remainder 5 is the Altitude of the Crown.

To find CD, the Diameter of the Bottom of the Crown.

Meaſure A B, the Diameter of the Top, which, admit it be 99 Inches; then hold a Thread ſo as a Plumbet at the End thereof may hang juſt over C, by which means you will find the Diſtance A a. Do the like on the other Side; ſo will you find alſo the Diſtance, B; which ſuppoſe 17.5 Inches each; add theſe two together, and ſubtract their Sum (*viz.* 35) from 99, and the Remainder is 64 Inches, the Diameter at the Bottom of the Crown. The Diameter which touches the Top of the Crown, may be found by the Sliding-rule to be 65 Inches.

Now to find the Content of the Copper from the Crown upwards (that is, the Part A B k h, the Depth gd being 42 Inches, you may take the Diameter in the Middle of every 6 Inches of the Depth, which ſuppoſe to be as in the ſecond Column of the following Table, the Numbers in the third Column are the reſpective Areas in Ale Gallons, found by Problem III. the Fourth Column ſhews the Content of every 6 Inches; all which being added together, the Sum will be the Content of that Part, ABkh; that is, ſo much as it will hold after the Crown is cover'd.

Now, if the Crown be taken for the Fruſtum of a Sphere, the Content (by the latter Part of Sect. II. Page 190.) will be found to be 28.75 Gallons.

But may be more readily found, very near the Truth, thus:

The Diameter C D, was found to be 64, and the Area to this Diameter is 11.408; this, multiply'd by half the Crown's Altitude, *viz.* by 2.5, gives 28.52 Gallons, the Content of the Crown.

The Content of the Part hkDC is 57.935 Gallons; from which ſubtract the Content of the Crown, 28 52, and the Remainder is 29.415 Gallons, and ſo much Liquor will juſt cover the Crown.

Parts

Parts of the Depth	Diameter.	Areas.	Content of every 6 Inches.
6	95.3	25.2945	151.767
6	90.1	22.6095	135.657
6	85.0	20.1223	120.734
6	80	17.8246	106.947
6	75.2	15.7499	94.499
6	70.5	13.8426	83.056
6	66	12.1319	72.791
The Sum - - -			765.451
To juſt cover the Crown			29.415
The whole Content - -			794.866

By Scale and Compaſſes.

You may find the Areas anſwering to every one of the Diameters, thus:

Extend the Compaſſes from the Gauge-point to the Diameter; that Extent, being turn'd twice over from 1, will at laſt fall upon the Area of that Circle: Or, being turn'd twice over from 6, will give the Content of that 6 Inches of the Depth.

Example. Extend the Compaſſes from 18.95 (the Gauge-point) to 95 3; that Extent, turn'd twice over from 6, will at laſt fall upon 151.76 Gallons, the Content of the firſt 6 Inches. And ſo of the reſt.

Problem

PROBLEM X.

To compute the Content of any Cloſe Caſk.

IN order to perform this difficult Part of Gauging, the three following Dimenſions of the Caſk muſt be truly taken;

$$\textit{Viz}\left\{\begin{array}{l}\text{The Bung-diameter,}\\ \text{The Head-diameter,}\\ \text{The Length of the Caſk,}\end{array}\right\}\text{within the Caſk.}$$

In taking theſe Dimenſions, it muſt be carefully obſerv'd,

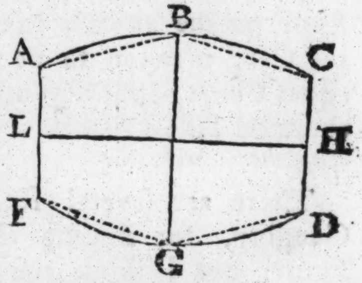
1. That the Bung-hole be in the Middle of the Caſk; alſo, that the Bung-ſtaff, and the Staff oppoſite to the Bung-hole, are both regular and even within.

2. That the Heads of the Caſk are equal, and truly circular; if ſo, the Diſtance between the Inſide of the Chine to the Outſide of its oppoſite Staff will be the Head-Diameter within the Caſk, very near.

3. With a ſliding Pair of Calipers (made for that Uſe) take the ſhorteſt Diſtance, or Length, between the Outſides of the two Heads; from that Length ſubtract $1\frac{1}{5}$ Inch (more or leſs, according to the Largeneſs of the Caſk) for the Thickneſs of the Head: The Remainder will be the Length of the Caſk within. But if the Caſk be empty, you may take the Length, by putting a ſtrait Rod in at the Tap-hole, and allow for the Thickneſs of the Head.

Now by theſe Dimenſions, one would think the Content of the Caſk was perfectly limited; but it will be eaſy to perceive, by the following Figure, that the Diameters and Length of one Caſk may be equal to thoſe of another, and yet one of thoſe Caſks may contain ſeveral Gallons more than the other.

As for Inſtance, the Figure ABCDF is ſuppos'd to repreſent a Caſk: Then it is plain, that if the outward curve Lines, ABC, and FGD, are the Bounds or Staves of the Caſk, it muſt needs hold more than if the inner prick'd Lines were the Bounds, or Staves; and yet the Bung-diameter BG, and Head-diameters CD and AF, and Length LH, are the ſame in both thoſe Caſks.



Whence it appears, that no one general Rule can be given, whereby the Content of all Sorts of Caſks can be gauged: and therefore Gaugers do uſually ſuppoſe every Caſk to be in ſome of theſe Forms:

1. The middle Fruſtum of a Spheroid.
2. The middle Fruſtum of a parabolic Spindle.
3. The lower Fruſtums of two equal parabolic Conoids.
4. The lower Fruſtums of two equal Cones.

1. If the Staves of a Caſk be very much curved (as the outward Lines of the laſt Figure), then the Caſk is ſuppoſed to be the middle Fruſtum of a Spheroid.

2. If the Staves (between the Bung and the Head) be ſomething leſs curved, then the Caſk is taken to be the middle Fruſtum of a parabolic Spindle.

3. If the Staves (between the Bung and Head) be very little curved, then the Caſk is taken to be the lower Fruſtums of two equal parabolic Conoids, abutting,

ting, or joining together, upon one common Baſe.

4. If the Staves (between the Bung and Head) be ſtrait (as the prick'd Lines in the laſt Figure), then the Caſk is taken to be the lower Fruſtums of two equal Cones, abutting or joining together upon one common Baſe.

There are ſeveral Rules laid down in Books of Gauging, for finding the Content of each ſeveral Form; but I think the ſhorteſt and moſt practical Way is, to find ſuch a mean Diameter, which will reduce the propoſed Caſk to a Cylinder; Thus,

Multiply the Difference of the Bung and Head Diameters by .7 for the Spheroid; by .65 for the ſecond Form, by .6 for the third Form, and by .55 for the fourth Form; and add the Product to the Head-diameter, and the Sum is a mean Diameter.

Example. Suppoſe the Bung-diameter be 32 Inches, the Head-diameter 24 Inches, and the Length 40 Inches; the Content in each Variety is required.

The Difference between the Bung and Head-diameter is 8; which multiply'd by .7, the Product is 5.6; which added to the Head-diameter, the Sum is 29.6, the mean Diameter: The Area anſwering thereunto will be found (by Prob III.) to be 2.44 Ale Gallons; which multiply'd by the Length, the Product is 97.4 Gallons; and ſo much is the Content, if it be the firſt Form.

Again, if the Difference of the Diameters 8 be multiply'd by .65, the Product will be 5 2; which added to the Head-diameter, the Sum is 29 2, for the mean Diameter; and the Area anſwering thereunto is 2.3746 Gallons; which multiply'd by 40 (the Length), the Product is 94.98 Gallons, the Content, if it be of the ſecond Form.

Again, if the Difference 8 be multiplied by 6, the Product is 4.8; which added to the Head-diameter, the Sum is 28.8, the mean Diameter; the Area thereunto is 2.31 Gallons; which, multiply'd by 40, gives the Content 92.4 Gallons, for the third Form.

Again, the Difference 8, multiply'd by .55, the Product is 4.4; which added to the Head-diameter, makes the mean Diameter 28.4; the Area thereof is 2.2463; which multiply'd by 40, the Product is 89.85 Gallons, for the fourth Form.

By Scale and Compaſſes.

Extend the Compaſſes from the Gauge-point 18.95, to the firſt mean Diameter 29.6; that Extent will reach from the Length 40, to a fourth Number, and then to the Content, 97.4 Gallons.

Again, Extend from 28.95 to 29.2 (the ſecond mean Diameter) that Extent, turn'd twice over from 40, will at laſt fall upon 94.98 Gallons.

Again, Extend from 18.95 to 28.8 (the third mean Diameter) that Extent, turn'd twice over from 40, will at laſt fall upon 92.4 Gallons.

Again, Extend from 18.95 to 28.4 (the fourth mean Diameter) that Extent, turn'd twice over from 40, will at laſt fall upon 89.85 Gallons.

Altho' I have all along made uſe of the Line of Numbers upon the common Two Foot, or Eighteen Inch Rules, for the Reaſon mentioned in the *Preface*; yet the Rules may eaſily be applied to the Sliding-rule, thus: To find the Area of a Circle in Gallons, ſet the Gauge-point upon D (that is, a ſingle Line of Numbers to 1 upon C (that is, a double Line);

then againſt any Diameter upon D, is the Area upon C, thus:

To find the Content of the Caſk, laſt mention'd, the firſt Form.

Set the Gauge-point 18.95 upon D, to the Length 40 upon C; then (againſt the mean Diameter) 29.6 upon D, is 97.4 Gallons, the Content upon C.

And againſt 29.2 (the next mean Diameter on D, is 94.98 Gallons on C.

And againſt 28.8 (the next mean Diameter) on D, is 92.4 Gallons on C.

And againſt 28.4 (the laſt mean Diameter) on D, is 89.85 Gallons on C.

All done without removing the Slider.

A TABLE *of the Segment of a Circle, whose Area is Unity.*

V. S.	Segm	V. S.	Segm	V. S.	Segm	V. S.	Segm
1	.0017	99	.9983	26	.2066	74	7924
2	0048	98	.9952	27	.2178	73	.7832
3	0087	97	.9913	28	.2292	72	.7708
4	0134	96	.9866	29	.2407	71	7593
5	0187	95	.9813	30	.2523	70	.7477
6	0245	94	.9755	31	.2640	69	.7360
7	0308	93	.9692	32	.2759	68	.7241
8	0375	92	.9625	33	.2878	67	.7122
9	.0446	91	.9554	34	.2998	66	.7002
10	0520	90	.9480	35	.3119	65	.6881
11	.0598	89	.9402	36	3241	64	.6759
12	.0680	88	.9320	37	.3364	63	.6636
13	0764	87	.9236	38	.3487	62	.6513
14	.08[illegible]1	86	.9149	39	3611	61	.6389
15	0941	85	.9059	40	3735	60	.6265
16	.1033	84	.8967	4	.3860	59	.6140
17	.1127	83	.8873	42	.3986	58	.6014
18	.1224	82	.8776	43	.4112	7	.5888
19	.1323	81	.8677	4	.4238	56	.5762
20	.1424	80	.8576	45	4364	55	.5636
21	.1526	79	.8474	46	.4491	54	.5509
22	.1631	78	.1369	47	.4618	53	.5382
23	.1737	77	.8263	48	.4745	52	.5255
24	.1845	76	.8155	49	.4873	51	.5127
25	.1955	75	.8045	50	.5000	50	.5000

The Uſe of the Table of Segments

Is to find the Ullage, or Quantity of Liquor remaining in a Caſk, whoſe Axis is parallel to the Horizon, the Surface of the Liquor cutting the Heads of the Caſk.

The RULE *is*;

To the wet or dry Inches of the Bung-diameter, add a competent Number of Cyphers; then divide it by the whole Diameter, the Quotient found in the Table under the Title V. S. gives a Segment; which multiply'd by the whole Content of the Caſk, the Product ſhews the Quantity of Liquor in the Caſk, if the Dividend was the wet Inches, or the Ullage, if it was the dry.

Let there be a Caſk in Form of a Cylinder, whoſe Bung-diameter is 29 Inches, the dry Part 13, and the wet 16, and the Content 80 Gallons; How many Gallons are wanting to fill the Caſk?

Divide the dry Inches, 13 by 29, the Bung-diameter, and the Quotient is .448; find the two firſt Figures .44 under V. S. and the Segment againſt it is .4238; to which add a proportional Part for the 8, and the whole Segment will be .4333; which multiply'd by the Content of the Caſk, the Product will be 34.664 Gallons; and ſo much the Caſk wants of being full.

Note, If the Caſk be in the Form of a Cylinder, or near that Figure, the Table will give the Ullage exact enough; but if it be a ſpheroidal Caſk, then uſe the following Method.

1. By the Bung and Head-diameter, find ſuch a mean Diameter as, you judge, will reduce the propos'd Caſk to a Cylinder, and then find its Content.

2. From the Bung-diameter ſubtract the mean Diameter, and take half the Difference.

3. From the wet Inches ſubtract the ſaid Half-difference; reſerve this Difference, then uſe this Proportion:

As the mean Diameter is to 100
(the Diameter of the tabular Circle),
ſo is the reſerv'd Difference,
to a verſed Sine in the Table.

Then, if the tabular Segment be multiply'd into the Content (as before) the Product will be the Quantity of Liquor in the Caſk.

Example. Let the Caſk be the ſame as in Page 325, of the firſt Form, where the Bung-diameter is 32 Inches, and the mean Diameter 29.6, and the Content 97.4 Gallons; and ſuppoſe the wet Inches 19, to find the Quantity of Liquor in the Caſk.

From 32 From 19
Subtr. 29.6 Subtr. 1.2

Rem. 2.4 Rem. 17.8 reſerv'd.

Half 1.2

29.6 : 100 : : 17.8 : .60, the V. S.

The Segment to 60 is .6265, which multiply'd by 97.4 the Content, the Product is 61 Gallons, the Quantity of Liquor in the Caſk.

If the dry Inches had been given, by the ſame Method, you might have found the Ullage, or what the Caſk wanted of being full.

To find what Quantity of Liquor is in a Caſk, when its Axis is perpendicular to the Horizon, *viz.* when it ſtands upright upon one of its Heads.

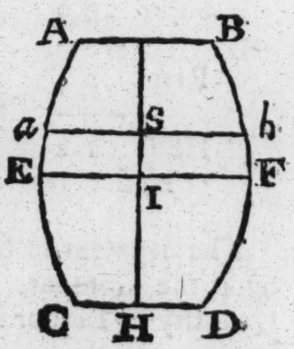
To do this, you muſt know how to calculate the Area of any Circle, between the Bung and Head, whoſe Diſtance from the Bung, or Middle of the Caſk, is given; which may be done by this Proportion.

As the Square of half the Length of the Caſk is to the Difference between the Bung and Head-areas; ſo is the Square of any Circle's Diſtance from the Bung, to the Difference between the Bung-area and the Area of that Circle, *viz.* the Area of the Liquor's Surface.

Then, from the Bung-area, ſubtract one-third Part of the aforeſaid Difference, *viz.* between the Bung-area and the Area of the Liquor's Surface: Multiply the Remainder by the Liquor's Diſtance from the Bung, and the Product will ſhew what Quantity of Liquor is either above or under half the Content of the Caſk.

Example. Let us again ſuppoſe the Caſk, in Page 325, whoſe Length is 40 Inches, Bung-diameter 32, and Head-diameter 24, and ſuppoſe the wet Inches, SH, 26 Inches.

The Sq. of half the Length is 400, the Diſtance of the Liquor's Surface from the Bung SI is 6, whoſe Square is 36; the Area of the Bung D 2.8519 Ale-Gallons, and the Area



Area of the Head D 1.6042; the Difference 1.2477. Then,

400 : 1.2477 : : 36 : .0751

One-third is = .0250

From	2.8519	Bung-area,
Subtr.	.0250	A Third of the Difference.
Rem.	2.8269	
	6	Mult. Diſt. from the Bung.
	16.9614	Content above the Bung.
Add	48.7	Half the Content of the Caſk.
	65.66	The Quantity of Liquor in the G.

* * *

PROBLEM XI.

Gauging of MALT.

TO find the Quantity of Malt in a Ciſtern, or upon a Floor,

Firſt, Find the Area of the Baſe in Buſhels, by multiplying the Length by the Breadth, and dividing the Product by 2150.42, or only by 2150; and multiply that Area by the mean Depth (how to take the mean Depth, ſee Problem II.). If the Baſe be circular or oval, divide by 2731 (ſee Problem I).

Example. There is a Ciſtern, whoſe Length is 84 Inches, and Breadth 54 Inches, and the mean Depth is 43.6 Inches; What is the Content?

Multiply 84 by 54, and the Product is 4536; which divide by 2150, and the Quotient is 2.1097 Buſhels, the Area of the Bottom at 1 Inch deep; which multiply'd by the Depth 43.6, the Product is 91.98 Buſhels, the Content.

Example. Suppoſe a Quantity of Malt upon a Floor, whoſe Length is 245 Inches, and the Breadth 184 Inches, and the mean Depth 5.6 Inches; How many Buſhels are there?

Multiply 245 by 114, and the Product is 45080; which divided by 2150, the Quotient is 20.967, the Area of the Baſe; which multiply'd by the mean Depth, the Product 117.4 Buſhels, the Content.

By the Sliding Rule.

There is an inverted Line of Numbers upon ſome Sliding-rules, mark'd with the Letter M, which was contriv'd purpoſely for Gauging of Malt; and there is a double Line of Numbers upon the Rule, and upon the Slider two double Lines of Numbers; all of theſe are of equal Radius, and all work together at once: Thus ſet the Length and Breadth againſt one another upon the inverted Line, and that which ſlides by it; then, on the other Edge of the Rule againſt the Depth, you will find the Content in Buſhels. Thus, in the firſt Example, ſet 54 upon the Slider againſt 84, upon the inverted Line; and then, againſt 43.6 upon the other Part of the Rule, is 91.98 upon the Slider.

Again, in the ſecond Example, ſet 184 upon the Slider to 245 upon the inverted Line; and againſt 5.6 upon the other Part of the Rule, is 117.4 upon the Slider.

§ II.

§ II. *Of* LAND-MEASURING.

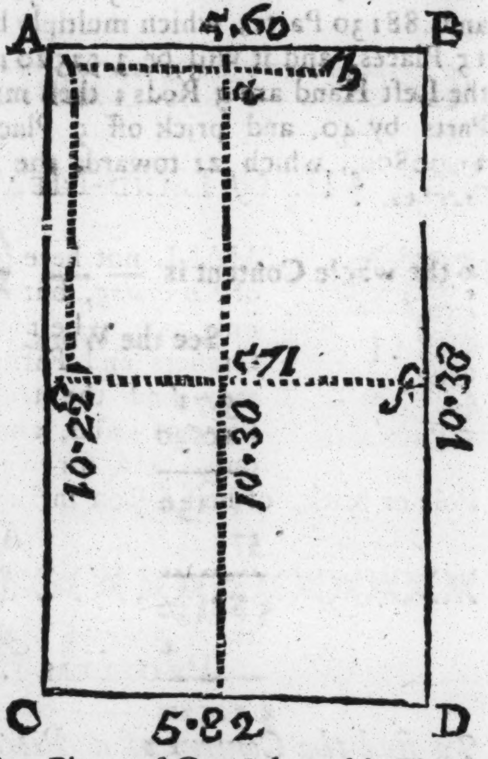
I SHALL not here give the whole Art of *Surveying*, but ſuch practical Rules only as may be uſeful to the Country Graſiers and Farmers, whereby they may find the true Content of any Piece of Land, and that by the Chain only (and for want of that, with a Pole or Stick, of half a Rod in Length).

⁂

PROBLEM I.

To find the Content of a Piece of Land in the Form of a right-angled Parallelogram, or long Square, or what is ſomething near that Form.

TO know whether any Angle in a Field be a Right-angle, or not, you may take a Piece of Board about 4 or 5 Inches broad, and an Inch thick, either round or ſquare; and, with a Saw, cut two Kerfs, croſſing each other at Right angles; and bore a Hole in the Middle of the Back-ſide, to put it upon the End of a Stick. This will repreſent the Inſtrument call'd a Croſs.



Suppoſe you would obſerve the Angle A, to know whether it be a Right angle (or near thereunto); prick up your Stick, with the Croſs upon it, a little Diſtance from the Fence, as at *a*; and having ſet up two Marks, as at *b* and *c*, of equal Diſtance from the Fence, turn one of the Slits directly towards *b*; and then, if the other be directly pointing to *c*, it is a Right-angle.

To meaſure ſuch a Piece of Ground as this Figure above: If you meaſure round, and add the oppoſite Sides together, and take half the Sum (if they be not equal); or elſe meaſure down about the Middle of the Length, and Middle of the Breadth; thus, the Side AB being meaſured, it will be 5.60 (that is, 5 Chains and 60 Links); and the oppoſite Side C D is 5 Chains 82 Links; the half Sum thereof is 5.71: And the Side BD is 10.38; and the Side AC 10.22; and the half Sum thereof is 10 30 (it will be the ſame thing, if you meaſure about the Middle of the Length, and Middle of the Breadth); then multiply this mean Length and mean Breadth together, *viz.* 10.30, by 5.71, and the Product is 58.8130; which divide by 10 (becauſe 10 Square Chains is an Acre) by removing the ſeparating Point one Place towards the Left-hand

Left Hand, and it will be 5.88130; that is, 5 Acres and .88130 Parts; which multiply by 4, and prick off 15 Places, and it will be 3 52520; which 3 towards the Left Hand are 3 Rods; then multiply the decimal Parts by 40, and prick off 5 Places, and it will be 21.00800; which 21 towards the Left Hand are 21 Perches.

	A	R.	P.
So the whole Content is — —	5	3	21

See the Work.

5.71
10.30
17130
571 A. R. P.
5.88130 5 3 21
4
3.52520
40
21.00800

Note, The Chain here made uſe of, is 4 Poles, or Rods, in Length; the whole Chain being 100 Links.

But, becauſe every Man that may have Occaſion to meaſure a Piece of Land, can't procure a Chain, I will therefore ſhew how you may meaſure a Piece of Land only with a Stick of half a Rod in Length; that is, 8 Feet and 3 Inches; which Stick divide into five equal Parts, ſo will the whole Rod be divided into ten Parts, and will be thereby adapted to Decimal Arithmetick.

But, becauſe each of thoſe Parts of the Stick are ſomething large (each Part being 19 Inches and 8 Tenths) it will be neceſſary to take your Dimenſions to half of one of thoſe Parts; and then, ſor that half Part, ſet 5 in the Place of Seconds, thus; ſuppoſe 3 Parts and a half, ſet it down thus .35.

* * *

PROBLEM II.

LET us ſuppoſe a Field in the Form of a long Square, whoſe Length is 45 Rods 5 Parts and a Half, and the Breadth 31 Rods 4 Parts and a Half: What is the Content?

Multiply the Length and Breadth together, and divide the Product by 160 (becauſe 160 Square Rods is an Acre) and the Quotient is Acres.

```
     45.55
     31.45
    ------
     22775
    18220
    4555
  13665
  --------
  1432.5475
```

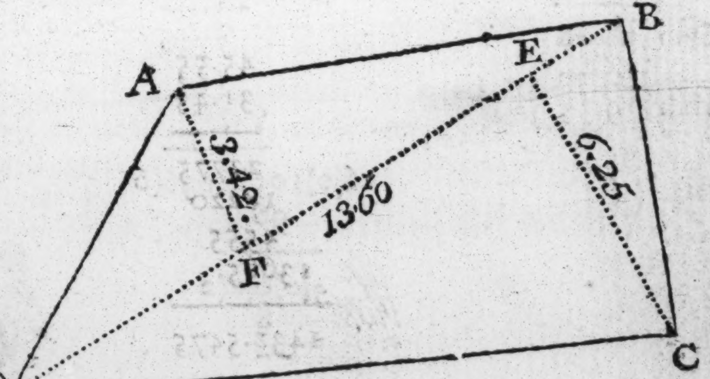
160

```
16|0)143|2(8       A. R. P.
     128     Facit 8  3  32
     ---
 4|0) 15 2(3
      12
     ---
      32
```

* * *

Problem III.

SUppoſe a Piece of Ground in the Form of a Trapezium; the Diagonal B D 13 Chains 60 Links, the Perpendicular CE 6 Chains 25 Links, and the Perpendicular A F 3 Chains 42 Links; What is the Content?



Multiply the Diagonal by half the Sum of the Perpendiculars. See Sect. VI. of Chap. I. Part II.

CE=6.25 13.60=BD
AF=3.42 4.83

Sum 9.67 4080
10880 A. R. P.
Half 4.83 5440 *Facit* 6 2 11

6.56880
4

2.227520
40

11.00800

By Rods, thus;

CE=25 Rods. 19.34
AF=13.68 544=BD

Sum 38.68 7736
7736
Half 19.34 9670

16,0)105|2.096(6
96

4|0)9|2(2
8

12

A. R. P.
Facit 6 2 12

To

To take the Dimenſions of the Field.

Begin at the Angle B, and meaſure in a direct Line towards D; but when you come at E, ſet up your Croſs, and direct one of the Slits to D, and then look through the other Slit, and if it exactly hits the Angle C, then are you juſt in the Place where the Perpendicular will fall; but if it does not exactly hit the Point, move backwards and forwards till it does ſo; then meaſure the Perpendicular, and ſet down the Chains and Links, or the Rods and Parts; then continue your Meaſure towards D; but when you come to F, ſet up your Croſs, and try (as is above directed), whether you be in the Place where the Perpendicular will fall. Then meaſure the Perpendicular A F, and ſet down the Chain and Links or Rods and Parts; then continue your Meaſure to D, and ſet down the Meaſure of the whole Diagonal. This Way of Meaſuring is very exact and true; but the common Way uſed by the Graſiers and Farmers, is to meaſure round the Field, and to take half the Sum of the oppoſite Sides for a mean Side; but the laſt-mentioned Piece of Ground, being meaſur'd ſo, will come to

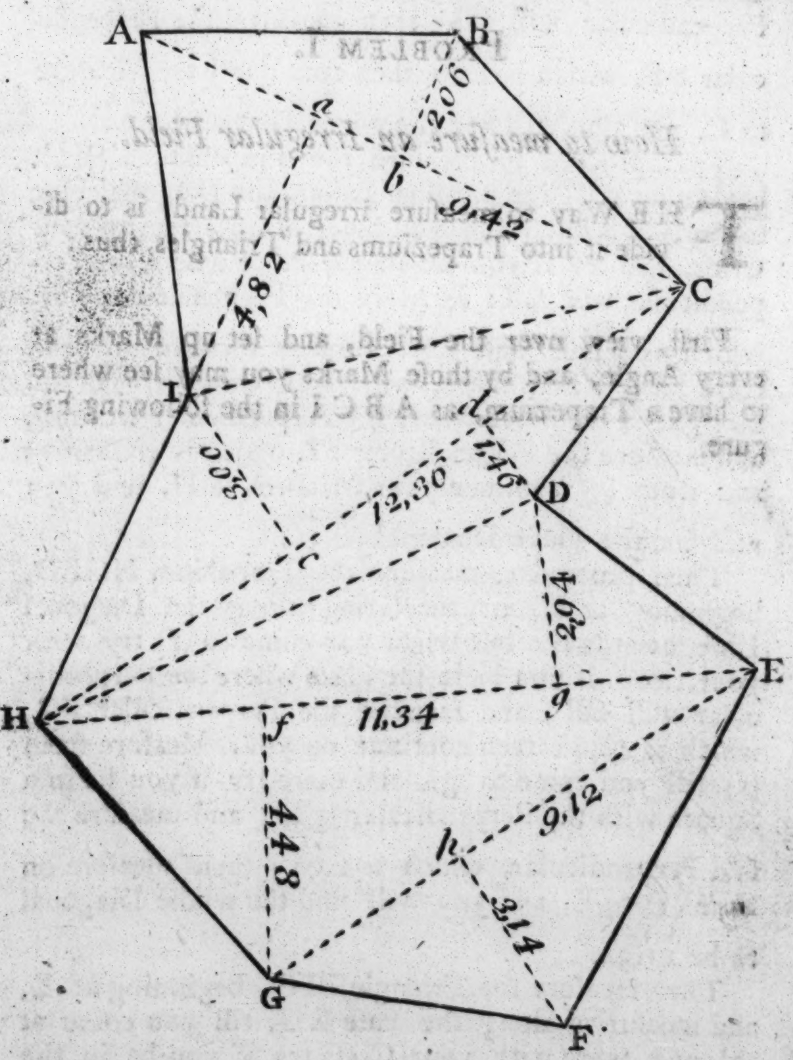
A. R. P. R. P.
7 0 22, which is 2 10 more than the Truth.

PROBLEM I.

How to meaſure an Irregular Field.

THE Way to meaſure irregular Land, is to divide it into Trapeziums and Triangles, thus:

Firſt, view over the Field, and ſet up Marks at every Angle, and by thoſe Marks you may ſee where to have a Trapezium, as A B C I in the following Figure.



Then begin and meaſure in a direct Line from A towards C; but when you come to (a), ſet up your Croſs, and try whether you be in a Square to I (as is before directed); and then meaſure the Perpendicular a I, which is 4.82; then meaſure forward again towards C, but when you come to (b) ſet up your

Croſs, and try whether you be in the Place where the Perpendicular will fall; then meaſure the Perpendicular b B, which is $\overset{\text{Ch. L.}}{2.06}$ then continue your Meaſure to C, and you will find the whole Diagonal $\overset{\text{Ch. L.}}{9.42}$.

Then proceed to meaſure the Trapezium CDHI, beginning at C, and meaſuring along the Diagonal Line towards A; but when you come at (d), ſet up your Croſs, and try if you be in the Place where the Perpendicular will fall: Meaſure the Perpendicular d D, which is $\overset{\text{Ch. L.}}{1.46}$, and then meaſure forward till you come at (c), and there, with your Croſs, try if you be right in the Place where the Perpendicular will fall, and meaſure the Perpendicular c I, which is 3 Chains; and from (c) continue your Meaſure to H, and you will find the whole Diagonal $\overset{\text{Ch. L.}}{12.36}$.

Then proceed to meaſure the Trapezium HGED, beginning at H, and meaſuring along the Diagonal Line towards E; but when you come to (f) try with your Croſs, if you be in the Place where the Perpendicular will fall; and meaſure the Perpendicular f G, which is 4.48; then continue on your Meaſure from (f) till you come to (g), and there try if you be in a Square with the Perpendicular g D; and meaſure the ſaid Perpendicular, which is $\overset{\text{Ch. L.}}{2.94}$; then meaſure on from (g) to E, and you will find the whole Diagonal to be $\overset{\text{Ch. L.}}{11.34}$.

Then meaſure the Triangle EFG, beginning at E, and meaſuring along the Baſe E G, till you come at (h), and there with your Croſs try if you be in the Place where the Perpendicular will fall; and meaſure the Perpendicular h F, which is $\overset{\text{Ch. L.}}{3.14}$, continue your Meaſure to G, and you will find the whole Baſe to be $\overset{\text{Ch. L.}}{9.12}$; ſo you have finiſhed your whole Field.

I have been the larger upon the Explanation of this Problem, becauſe moſt Grounds lie in ſuch irregular Forms.

Caſt up the three Trapeziums ſeverally, and alſo the Triangle; and add all the ſeveral Areas together into one Sum, which will be the Area of the whole irregular Plot.

See the Work.

b B=2.06	9.42	See Sect. VI. Chap. I.
a I=4.82	3.44	Part II.
Sum 6.88	3768	
	3768	
Half 3.44	2826	
	3.24048=Area of ABCI,	
dD=1.46	12.36	
c I=3.00	2.23	
Sum 4.46	3708	
	2472	
Half 2.23	2472	
	2.75628=Area of CIHD.	
fG=4.48	11.34	
gD=2.94	3.71	
Sum 7.42	1134	
	7938	
Half 3.71	3402	
	4.20714=Area of HGED.	

Baſe=9.12

Half=4.56 See Sect. V. Chap. Part II.

Perpend. 3.14

1824

456

1368

1.43184=Area of the Triangle EFG.

3.24048=Area of ABCI.

2.75628=Area of CIHD.

4.20714=Area of HGED.

Sum 11.63574=Area of the Whole.

4

2.54296

40

21.71840

A. R. P.

Facit 11 2 21

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